

**FINITE ELEMENT SIMULATION OF CREEP IN  
SWOLLEN NITRILE BUTADIENE RUBBER (NBR)  
PIPE UNDER INTERNAL PRESSURE**

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## ABSTRACT

*Elastomers are increasingly used as engineering materials due to their high manufacturability, low weight, low cost and excellent mechanical properties. During the service, elastomer degradation can occur as the result of interaction with the environment. In the case of exposure to aggressive liquids or solvents such as palm biodiesel, degradation in the form of swelling can occur which leads to a reduction in its properties. Nitrile Butadiene Rubber (NBR) is one type of elastomers which is widely used where high oil resistance is required such as in automotive seals, gaskets, or other items subject to contact with oils.*

*The objective of this dissertation is to investigate the effect of swelling on the creep behavior of elastomeric pipe under internal pressure using finite element simulation. For this purpose, the commercial finite element code ANSYS is used to simulate the creep response of dry and swollen NBR. Two swelling levels are considered. During simulation, both dry and swollen elastomers are assumed to be visco-hyperelastic, isotropic and incompressible. The shear relaxation modulus of elastomers is obtained from experimental works conducted by other students. The results showed significant creep on both dry and swollen elastomers. Moreover, it is observed that the creep decreases as the swelling level increases. These results are consistent with the experimental finding in the literature.*

## ABSTRAKT

*Elastomer semakin kerap digunakan sebagai bahan kejuruteraan kerana sifat ringan, berkos rendah, mudah dibuat dan mempunyai sifat-sifat mekanik yang baik. Sepanjang digunakan, degradasi elastomer boleh berlaku akibat interaksi dengan persekitaran. Sekiranya ia terdedah kepada cecair yang agresif atau pelarut seperti biodiesel sawit, degradasi dalam bentuk bengkak boleh berlaku yang membawa kepada penurunan dalam sifat-sifatnya. Getah Nitril Butadiena (NBR) adalah salah satu jenis elastomer yang digunakan secara meluas di dalam keadaan yang memerlukan rintangan minyak yang tinggi diperlukan seperti dalam seal automotif, gasket, atau barangan lain yang bersentuhan dengan minyak.*

*Objektif disertasi ini adalah untuk mengkaji kesan bengkak pada kelakuan rayapan paip elastomerik bawah tekanan dalaman dengan menggunakan simulasi unsur terhingga. Bagi tujuan ini, perisian unsur terhingga iaitu ANSYS digunakan untuk mensimulasikan tindak balas rayapan NBR kering dan bengkak. Dua tahap bengkak digunakan dalam simulasi ini. Semasa simulasi, kedua-dua elastomer kering dan bengkak dianggap sebagai Visco-hiperelastik, isotropik dan tidak mampat. Kelonggaran modulus ricih elastomer diperoleh dari eksperimen yang dijalankan oleh pelajar lain. Keputusan menunjukkan rayapan yang besar berlaku ke atas kedua-dua elastomer kering dan bengkak. Selain itu, dapat diperhatikan bahawa rayapan berkurang apabila kenaikan tahap bengkak. Keputusan ini adalah selaras dengan dapatan eksperimen dalam bahan rujukan terdahulu.*

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## ***CHAPTER1: Introduction***

### ***1.1 Overview***

Elastomer is one of the types of polymers. They are widely used owing to their properties like rebound, tensile strength, resistance to petroleum products, sustainability in cold & hot weather. Wide variety of elastomers like nitrile, EPDM, silicone, neoprene, HNBR, butyl, natural rubber, urethane rubber, fluorosilicone & fluorocarbon etc. are available today. Below mentioned elastomers are commonly used in automobile industry. Elastomers, example nitrile butadiene and acrylonitrile butadiene are being utilized in an increasing number of long term, load bearing applications, used under severe high-temperature, and high-pressure conditions.

The mechanical properties of polymers are time-dependent, in order to intelligently design a product for a specific applications, it is necessary to know how these materials will respond to stress and deformation well below the nominal yield stress or strain of the material which may be applied to the product for periods of years or decades.

Properties that are particularly important for extended service are creep resistance and stress relaxation. When an instantaneous load is applied to an elastic part, the resulting deformation can be roughly predicted by the tensile modulus of the elastic. Deformation then continues at a slower rate indefinitely until the part ruptures. Upon removal of the load, some portion of the part's original dimensions will be recovered. Some portion of the deformation will remain permanent. This time, temperature and load dependent deformation is called creep and it is a result of the viscoelastic nature of rubber materials. In the other words, creep occurs when a constant force is continuously applied on a component, causing it to deform gradually.

Elastomer degradation occurs as the result of a chemical reaction between the elastomer and the medium or by absorption of the medium into the elastomer. This attack results in a swelling of the elastomer and a reduction in its tensile strength. The temperature and concentration of the corrodent will determine the degree of deterioration. Unlike metals, elastomers absorb varying quantities of the material they are in contact with, especially organic liquids and bio fuel. This can result in swelling, and cracking, swelling can cause softening of the elastomer and in a lined vessel introduce high stresses and failure of the bond.

When loading is more than momentary, creep data must be considered for purposes of material selection and design. The creep modulus represents the modulus of a material at a specific stress level and temperature over a specified period of time. By substituting the time, temperature, load dependent creep or apparent modulus for the instantaneous modulus in appropriate design equations, creep can be predicted. In this thesis the creep on pipe made of swollen rubber under constant internal pressure has investigated.

### ***1.2 Objectives and Scopes of Study***

The objectives of the present dissertation are:

1. To simulate the creep response of dry and swollen nitrile butadiene rubber (NBR) pipes under constant internal pressure using finite element code ANSYS.
2. To investigate the effect of swelling on the creep response of NBR pipe.

In this dissertation, no experimental work was conducted. The data pertaining to the relaxation modulus of dry and swollen NBR were collected from experimental works of other students. Moreover, Finite Element Analysis (FEA) was conducted on three types of NBR:

1) Not Swollen or Dry      2) 2 days swollen or D2S      3) 5 days swollen or D5S

### ***1.3 Dissertation Lineament***

The dissertation encompasses a detailed study of the mechanical properties of rubbers with the focus on creep deformation. It is organized into five chapters which include introduction, literature review, research methodology, results and discussion, and conclusion. The followings give brief description of each chapter.

Chapter one is an introduction of this project. It gives an overview of dry and swollen rubbers and shows the importance of creep study.

Literature review of the project is discussed in chapter two. This chapter includes a general review on history and development of rubbers, outline background information about the material properties and theory applied to the creep behavior of rubbers. An in-depth analysis of the mechanical properties of the polymers – rubbers is also presented in this chapter.

Chapter three focuses on research methodology, where the finite elements are discussed. This chapter discusses the creep simulation for the nitrile Butadiene swollen pipe under specific constant internal pressure.

Chapter four presents a discussion of the experimental results from the creep test, simulation test.

Finally, chapter five concludes the research findings and highlights the problems encountered during the research study. This chapter also includes the features of the limitations as well as the weaknesses of the creep simulation and recommendations for future work.

## ***CHAPTER 2: LITERATURE REVIEW***

### ***2.1 Description of Elastomers***

#### ***2.1.1 History and Development***

The different properties of the elastomers (elastic polymers) are based on arrangement and synthesis of these monomers. Elastomers are increasingly used as engineering materials due to their high manufacturability, low weight, low cost and good mechanical properties.

The elastomer with the longest history of use is natural rubber, which is made from the milky sap, or latex, of the Hevea and other trees. Natural rubber is still an important industrial polymer, but it now competes with a number of synthetic elastomers, such as styrene-butadiene rubber and polybutadiene, which are derived from by-products of petroleum and natural gas. More than half of all rubber produced goes into automobile tires; the rest goes into mechanical parts such as mountings, gaskets, belts, and hoses, as well as consumer products such as shoes, clothing, furniture, and toys. Elastomers in general are used as shock absorbers because of their low modulus and high damping characteristics[1]. Due to their high damping characteristics, elastomers are increasingly used in applications that are subjected to shock, impact and vibrations. Thus, understanding the mechanical behavior of elastomers over a wide range of strain-rates and temperatures could significantly improve the design capabilities of such applications.

#### ***2.1.2 Structure of Elastomers***

The term 'elastomer' (from "elastic polymer") refers to any member of a class of polymeric

substances that possess the quality of elasticity, i.e., the ability to regain shape after deformation. Elastomers are the base material for all rubber products, both natural and synthetic, and for many adhesives.

Most elastomers are hydrocarbons; i.e., they are composed principally of carbon and hydrogen and their compounds. Some occur naturally—e.g., polyisoprene, which is formed in the latex of the rubber tree and is processed into natural rubber. Most elastomers, however, are produced synthetically from derivatives of petroleum and natural gas[2]. Monomers such as isoprene, butadiene, and butylene are subjected to various polymerization reactions in which they are built up into large molecules. In many cases other chemical elements or compounds are incorporated into the polymer in order to modify basic properties—e.g., chlorine in polychloroprene (neoprene) and sulfur in polyalkylene polysulfide (Thiokol), which contribute to the oil-resistance of these rubbers. Properties can also be modified by producing elastomers as copolymers, i.e., polymers made up of more than one type of monomer. Examples include nitrile rubber (an acrylonitrile-butadiene copolymer) and butyl rubber (a copolymer of isobutylene and isoprene). In another method, some elastomers are blended with various plastic polymers such as polypropylene or polystyrene; the resultant materials, known as thermoplastic elastomers, or TPR's (thermoplastic rubber), retain the resilience of rubber but, unlike others, can be remolded and reprocessed upon the application of heat (a property important in recycling).[3]

A elastomeric molecule consists of several thousand chemical repeating units, or monomers, linked together by covalent bonds. The assemblage of linked units is often referred to as the “chain,” and the atoms between which the chemical bonding takes place are said to make up the “backbone” of the chain.

In most cases elastomers are made up of carbon backbones—that is, chains of carbon (C) atoms linked together by single (C–C) or double (C=C) bonds. In theory, carbon chains are highly flexible, because rotation around carbon-carbon single bonds allows the molecules to take up many different configurations. In order to be made into useful rubber products, elastomeric materials must be subjected to various modifications. These include: strengthening of the material by cross-linking the polymer chains (for instance, by sulfur atoms in the process known as vulcanization); further strengthening by fillers such as carbon black; and treatment with chemicals that provide resistance to weathering and chemical attack[4]. For fabrication into adhesives, elastomers are often dissolved in organic solvents and treated with various other additives to improve their application, adhesion, and durability.

### **2.1.3 NBR**

Nitrile rubber, like other synthetic elastomers (elastic polymers), was a product of research that took place during and between the two world wars. A group of acrylonitrile-butadiene copolymers, given the name Buna N, was patented in 1934 by German chemists Erich Konrad and Eduard Tschunkur, working for IG Farben. Buna N was produced in the United States during World War II as GR-N (Government Rubber-Nitrile), and subsequently the group of acrylonitrile-butadiene elastomers became known as nitrile rubber.[5]

In the production of NBR, acrylonitrile ( $\text{CH}_2=\text{CHCN}$ ) and butadiene ( $\text{CH}_2=\text{CH}-\text{CH}=\text{CH}_2$ ) are emulsified in water and then polymerized (their single-unit molecules linked into large, multiple-unit molecules) through the action of free-radical initiators. The amount of acrylonitrile present in the final copolymer varies from 15 to 50 percent. With increasing acrylonitrile content the

rubber shows higher strength, greater resistance to swelling by hydrocarbon oils, and lower permeability to gases. At the same time, however, the rubber becomes less flexible at lower temperatures, owing to the higher glass transition temperature of Acrylonitrile (i.e., the temperature below which the molecules are locked into a rigid, glassy state).[5]

#### ***2.1.4 Application of NBR***

NBR owes its many applications to its special mechanical properties; however, on the other hand, the especially nonlinear mechanical properties make the analysis of NBR very difficult. It is necessary to take into account of the nonlinear stress—strain relations and incompressible behavior of such material, in addition to its vast amount of deformation. In the past, a great deal of work has been devoted to such incompressible hyperelasticity and its implementation into various finite element codes. Especially, the finite element method is popular to investigate the performance of elastomeric seals when designing a new geometry or analyzing an existing one [6]It is very useful to evaluate the stress and strain state, contact pressure on surfaces, friction force and so on. The main uses of NBR are in fuel hoses, gaskets, rollers, and other products in which oil resistance is required. It is also employed in textiles, where its application to woven and non-woven fabrics improves the finish and waterproofing properties. A hydrogenated version, abbreviated as HNBR, is also highly resistant to thermal and oxidative deterioration and remains flexible at lower temperatures.

Nitrile rubber is mostly used where high oil resistance is required, as in automotive seals, gaskets, or other items subject to contact with hot oils. The rolls for spreading ink in printing and hoses for oil products are other obvious uses. NBR is also employed in textiles, where its application to woven and nonwoven fabrics improves the finish and waterproofing properties. NBR is made in a hydrogenated version (abbreviated HNBR) that is highly resistant to thermal

and oxidative deterioration and remains flexible at lower temperatures. HNBRs are widely used in the automotive industry for a multitude of seals, belts and hoses because of its strength and retention of its properties after long term exposure to heat, oil and fuel. They are also used as sealing materials in oil exploration and its processing.

## 2.2 Mechanical characteristics of Elastomers

### 2.2.1 *Hyper elasticity and modulus for hyper elastic material*

Throughout this text we focus for simplicity on homogeneous (or homogenized) materials. A material is said to be homogeneous when the distribution of the internal structure is such, that every material point has the same mechanical behavior. On the other hand, in a heterogeneous material the strain-energy function will additionally depend on the position of the material point in the reference placement  $X$ .

Hence, the strain-energy density is a postulated scalar-valued function of one tensorial variable, namely the deformation gradient  $F$ . For convenience we require this function to vanish in the reference placement where  $F = I$ , i.e., the reference placement is stress free. From physical observations we conclude that the strain energy increases monotonically with the deformation,

$$W(I) = 0 \text{ and } W(F) \geq 0 \quad (2-1)$$

The strain energy function attains its global minimum for  $F = I$  at the stress free state. Moreover, let us require that an infinite amount of energy is necessary to expand a body infinitely and to compress a body to zero volume, respectively.

$$W(F) \rightarrow \infty \quad \text{as} \quad \det F \rightarrow \infty \quad (2-2)$$



$$W(F) \rightarrow \infty \quad \text{as} \quad \det F \rightarrow 0 \quad (2-3)$$

The strain-energy density  $W(F)$  and the resulting constitutive equation must, of course, fulfil some requirements which arise from mathematical theory as well as from the physical nature of the material under consideration.

### **2.2.1.1 Isotropy**

A special but very important class of materials is isotropic materials. From the physical point of view isotropic materials are materials without any preferred direction. In terms of material symmetry an elastic material is said to be isotropic if its symmetry group  $S \equiv SO(3)$ . It is said to be anisotropic otherwise.[7]

For isotropic materials the strain-energy function can be represented as a function of the invariants of the right Cauchy-Green tensor

$$W(C) = W(I_1^C, I_2^C, I_3^C) \quad (2-4)$$

$$I_1^C = \text{tr}(C)$$

$$I_2^C = \frac{1}{2} (\text{tr}(C^2) - \text{tr}(C)^2) \quad (2-5)$$

$$I_3^C = \det(C)$$

Note that this representation follows from the strain-energy function being invariant upon rotations and, thus, equation (2-5) may equivalently be written in terms of the invariants of the left Cauchy-Green tensor  $W(b) = W(Ib_1, Ib_2, Ib_3)$  or its related strain measures.

With similar arguments the strain-energy function of an isotropic material can be expressed as a function of the Eigen values of the right Cauchy-Green tensor

$$W(C) = W(\lambda_1^2, \lambda_2^2, \lambda_3^2) \quad (2-6)$$

Here we made use of the fact that the Eigen values of tensor  $C$ ,  $\lambda_\alpha^2$ ,  $\alpha = 1, 2, 3$ , are the squares of the Eigen values of tensor  $U$ ,  $\lambda_\alpha$ . Moreover, in isotropic materials the principal directions of stress tensor and work conjugate deformation tensor coincide.

In order to express the constitutive relation in terms of strain invariants we exploit the fact that the stress-strain relation is given by an isotropic tensor function;

$$QW(C)Q^T = W(QCQ^T) \quad (2-7)$$

which is not restricted to isotropic materials. An isotropic tensor function  $W(C)$  can explicitly be represented as:

$$\frac{\partial W(C)}{\partial C} = \alpha_1 I + \alpha_2 C + \alpha_3 C^2 \quad (2-8)$$

where the  $\alpha_i$ , are scalar coefficients (so-called response coefficients), which may be evaluated for each material law in terms of tensor  $C$ ,  $\alpha_i = \alpha_i(I_1^b, I_2^b, I_3^b)$  (2-9)

Equation (2-9) is known as Richter representation or first representation theorem for isotropic tensor functions. By some algebra (see, e.g., [8, 9]) it can alternatively be formulated as

$$\frac{\partial W(C)}{\partial C} = \hat{\alpha}_0 I + \hat{\alpha}_1 C + \hat{\alpha}_2 C^{-1} \quad (2-10)$$

which is known as the alternative Richter representation or second representation theorem for isotropic tensor functions. The fundamental message of these theorems is that the stress response on the straining of an isotropic material is uniquely determined by only three parameters.

### ***2.2.1.2 Elastic material models***

The stress-strain relation of an elastic material follows by equation (2-4) from a strain energy potential, which, of course, should map the physical properties for every specific material under consideration. Consequently there exists a huge number of strain-energy functions and corresponding constitutive theories. The aim of this section is to summarize some well established and frequently employed models for reference.

More sophisticated elastic models are required for organic materials. Some of them exhibit a nonlinear stress-strain behavior even at modest strains. More importantly, there is a wide range of polymers and also biological tissues which are elastic up to huge strains. These materials show complex (and very different) nonlinear stress-strain behaviors. Specific strain energy-functions are designed to account for these phenomena.

The typical example for a material undergoing large strains is natural rubber. Many polymers also show (above a critical temperature) a rubbery behavior – the response is elastic without much rate or history dependence. Polymers with a heavily cross-linked molecular structure (elastomers) are the most likely to behave like ideal rubber, but also soft biological tissue shows rubbery behavior. Besides being elastic, the following feature is typical of rubbery materials: the material strongly resists volume changes. The bulk modulus is comparable to that of metals. On the other hand, rubbery material is very compliant in shear; the shear modulus  $\mu$  is of orders of magnitudes smaller than the shear resistance of most metals. This observation motivates the modeling of rubbery materials as being incompressible, i.e., the volume remains constant during deformation,  $\det F = 1$ .

To assure incompressibility of an elastic material the strain-energy function is postulated to be of the form

$$W^{isochor} = W(C) + p(F - 1) \quad (2-11)$$

where  $p$  plays the role of a Lagrangian multiplier. By equation (3-4) follows for the first Piola-Kirchhoff stress tensor the relation

$$\mathbf{P} = P \mathbf{F}^{-T} + \frac{\partial W}{\partial \mathbf{F}} \quad (2-12)$$

and for the second Piola-Kirchhoff stresses and the Cauchy stresses :

$$\mathbf{S} = P \mathbf{F}^{-T} \mathbf{F}^{+T} + \mathbf{F}^{-T} \frac{\partial W}{\partial \mathbf{F}} = P \mathbf{C}^{-1} + 2 \frac{\partial W}{\partial \mathbf{F}} \quad (2-13)$$

$$\boldsymbol{\sigma} = P \mathbf{I} + \frac{\partial W}{\partial F} \mathbf{F}^T = P \mathbf{I} + F \left( \frac{\partial W}{\partial F} \right)^T \quad (2-14)$$

These relations illustrate that pressure  $p$  cannot be determined from the materials response but only from additional equilibrium equations and boundary conditions [10]. To account later for both, volume preserving as well as volumetric deformations, we decompose the deformation gradient into an isochoric (or deviatoric) part :

$\mathbf{F}^{isochor}$  and a volume related part  $\mathbf{F}^{vol} = \sqrt[n]{\det(\mathbf{F})} \mathbf{I}$

$$\mathbf{F} = \mathbf{F}^{isochor} \mathbf{F}^{vol} = \sqrt[n]{\det(\mathbf{F})} \mathbf{F}^{isochor} \quad (2-15)$$

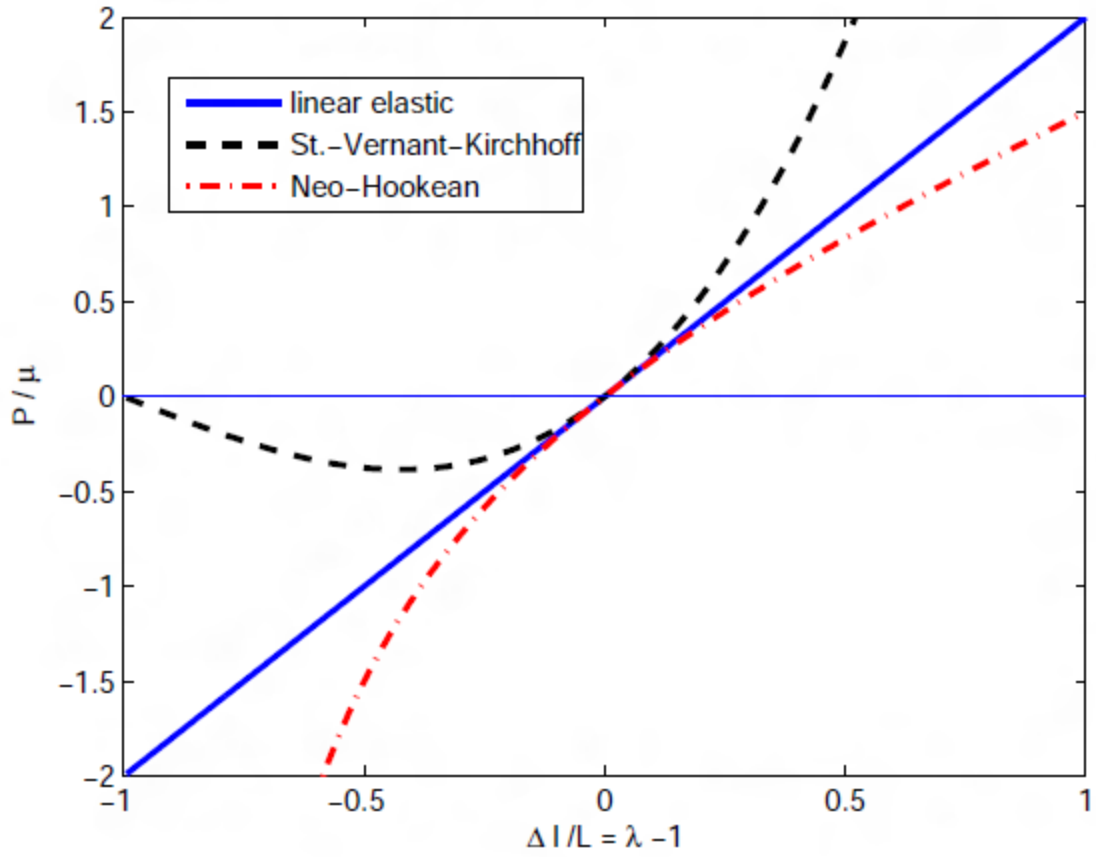


Figure 2. 1: Stress-Strain relations in uniaxial tension[11]

Let us now summarize the classical strain-energy functions for incompressible material ( $F^{isochor}$ ) but omit the superscript isochor for simplicity. As before,  $I_i^C$  denotes the i-th principle invariant of the (isochoric part of) tensor C, equation (2-10).

The simplest model is the Neo-Hookean solid,

$$W(C) = \frac{\mu}{2} (I_1^C - 3) \quad (2-16)$$

First used by Treloar [11], the parameter  $\mu$  was originally determined from an elementary statistical mechanics treatments predicting that  $\mu = \frac{N}{2}kT$  where N is the number of polymer chains per unit volume, k is the Boltzmann constant and T is the temperature. Today this model

is widely used with shear modulus  $\mu$  determined by experiments. The stress-strain relation follows from (2-12 , 2-14).

In Figure 2.1 the stress-strain relations in uniaxial tension are displayed. The red dash dotted line shows the Neo-Hookean model (2-16) whereas the black dashed line results from the Saint-Venant Kirchhoff model . In the undeformed placement the tangent on both curves is the straight line of the Hookean law . The limited validity of the Saint-Venant Kirchhoff is clearly visible. If  $\Delta l/L < -0.4226$  instability occurs, thence for rising compression a reduced stress is observed. Clearly, the model makes sense only for small compressive strains. (The critical strain value does not depend on the material data.). On the other hand, the Neo-Hookean model captures the physics for the full range of straining. From experiments we know that for rubbery materials under moderate straining up to 30 - 70 % the Neo-Hookean model usually fits the material behavior with sufficient accuracy.

To model rubber at high strains the one-parametric Neo-Hookean model (2-16) is meanwhile replaced by a more sophisticated development of Ogden [12, 13]. Instead of using strain invariants this model expresses the strain energy density in terms of principal stretches  $\lambda_\alpha$ ,  $\alpha = 1, 2, 3$ ,

$$W = \sum_{p=1}^N \frac{\mu_p}{\alpha_p} (\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3) \quad (2-17)$$

where  $N$ ,  $\mu_p$  and  $\alpha_p$  are material constants. In general, the shear modulus results from

$$2\mu = \sum_{p=1}^N \mu_p \alpha_p \quad (2-18)$$

The three principal values of the Cauchy stresses can be computed from (2-17) as

$$\sigma_\alpha = p + \lambda_\alpha \frac{\partial W}{\partial \lambda_\alpha} \quad \alpha = 1,2,3 \text{ (no summation)} \quad (2-19)$$

and the principal first and second Piola-Kirchhoff stresses follow by

$$p_\alpha = \lambda_\alpha^{-1} \sigma_\alpha \quad \text{and} \quad S_\alpha = \lambda_\alpha^{-2} \sigma_\alpha \quad (2-20)$$

With  $N = 3$  and values summarized in Table 2.1 the Ogden material reaches an almost perfect agreement to the experimental data of Treloar. Therefore and because it is computational simple, equation (2-17) is the reference material law for natural rubber.

Neo -Hookian	$\alpha_1 = 2.0$	$\mu = 4.225 \times 10^5 \text{ N/m}^2$
Ogden	$\alpha_1 = 1.3$	$\mu_1 = 6.3 \times 10^5 \text{ N/m}^2$
	$\alpha_2 = 5.0$	$\mu_2 = 0.012 \times 10^5 \text{ N/m}^2$
	$\alpha_3 = -2.0$	$\mu_3 = -0.1 \times 10^5 \text{ N/m}^2$
Moony - Rivilin	$\alpha_1 = 2.0$	$\mu_1 = 3.6969 \times 10^5 \text{ N/m}^2$
	$\alpha_2 = -2.0$	$\mu_2 = -0.5281 \times 10^5 \text{ N/m}^2$
Arruda - Boyce	$\lambda_{lock} = 3$	$\mu_0 = 3.380 \times 10^5 \text{ N/m}^2$
Baltz - Ko	$\vartheta = 0.45$	$\mu = 4.225 \times 10^5 \text{ N/m}^2$
St.Vernant - Kirchhoff	$\vartheta = 0.45$	$\mu = 4.225 \times 10^5 \text{ N/m}^2$

Table 2. 1: Material parameters

For particular values of material constants, the Ogden model (2-17) will reduce to either the Neo-Hookean solid ( $N = 1, \alpha = 2$ ) or the Mooney-Rivlin material. The Mooney-Rivlin material can be derived from (3-39) with  $N = 2$  and  $\alpha_1 = 2, \alpha_2 = -2$ , or, in other form

$$W(C) = \frac{\mu_1}{2} (I_1^C - 3) - \frac{\mu_2}{2} (I_2^C - 3) \quad (2-21)$$

together with equation (2-18). The Mooney-Rivlin material was originally also developed for rubber but is today often applied to model (incompressible) biological tissue, e.g. in [14-16].

In polymers or industrial rubbers the shear modulus  $\mu$  usually depends on the deformation.

Earlier as natural rubber these materials exhibit a rising resistance against straining. A physically inspired model for carbon filled rubber is the Arruda-Boyce model. It is also sometimes called the 8-chain model because it was derived by idealizing a polymer as 8 elastic chains inside a box [17]. This constitutive law has a strain energy density given by:

$$W(C) = \mu_0 \sum_{p=1}^N \frac{c_p}{\lambda_{lock}^{2p-2}} ((I_1^C)^p - 3^p) \quad (2-22)$$

Here,  $\mu_0$  is the (initial) shear modulus,  $c_p$  are constants following from statistical theory,  $\lambda_{lock}$  and  $N$  are material constants of the underlying chain model, namely the limiting chain extensibility and the number of rigid links, (see [18] for illuminating explanations). Evaluating the first three terms of expression (2-19) gives

$$W(C) = \mu_0 \left( \frac{1}{2} (I_1^C - 3) + \frac{1}{20\lambda_{lock}^2} ((I_1^C)^2 - 9) + \frac{11}{1050\lambda_{lock}^4} ((I_1^C)^3 - 27) \right) \quad (2-23)$$

In the example below the limiting chain extensibility is set to  $\lambda_{lock} = 3$  and the initial shear modulus is 80% of the Neo-Hookean shear modulus. The special feature of this model is a high strain resistance at strains  $> 300\%$  (controlled by the choice of parameters). In other words, the model has the ability to reflect the dependence of the resulting shear modulus on the deformation. Porous (or foamed) elastomers cannot be regarded as incompressible anymore. Blatz and Ko [19] proposed, basing on theoretical arguments and experimental data for polyurethane rubbers, a strain-energy density of the form

$$W(C) = \frac{\mu}{2} (I_1^C - 3) - \frac{\mu}{2\beta} ((I_3^C)^{-\beta} - 1) \quad (2-24)$$



where  $\beta$  is computed from shear modulus  $\mu$  and Poisson number  $\nu$  as  $\beta = \frac{\nu}{1-2\nu}$ . In the incompressible limit is  $I_3^C = 1$  and equation (2-24) reduces to the Neo-Hookean solid. Here the model is introduced because it is — either as Blatz-Ko model or as Neo-Hookean extended to the compressible range — applied for (porous) biological tissue, see e.g. [20, 21]

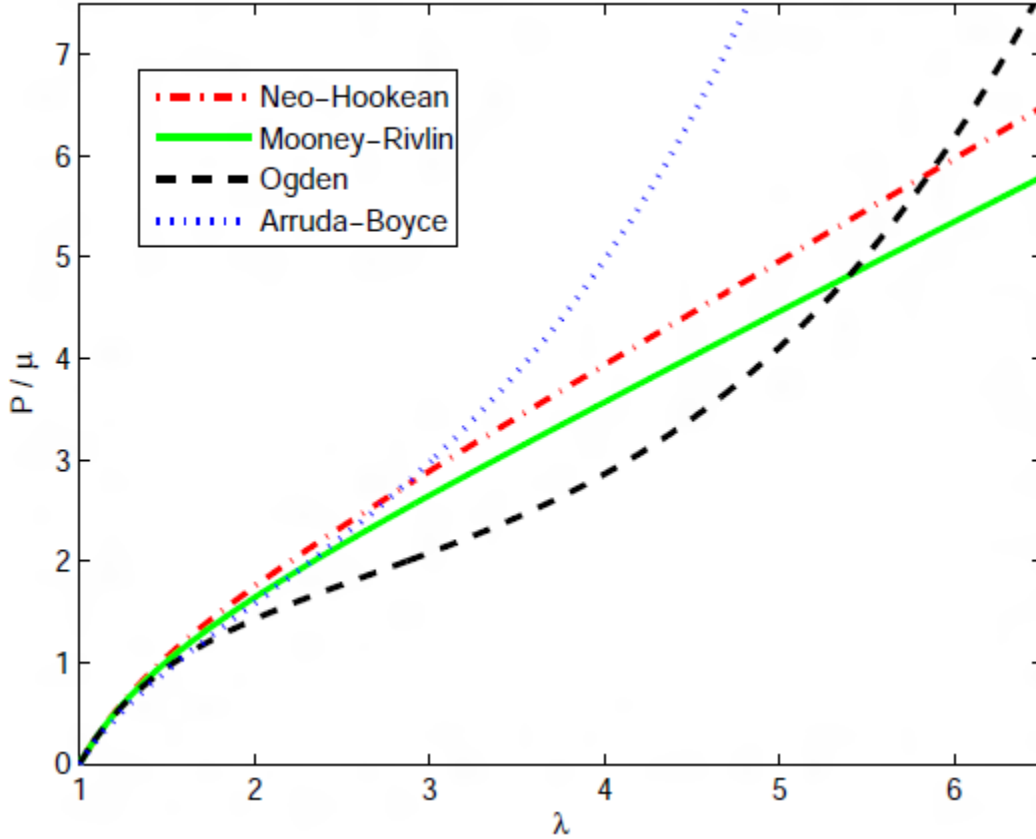


Figure 2. 2: Constitutive relations for rubbery materials in uniaxial tension[21]

Exemplarily, let us now consider an incompressible material under uniaxial tension. In particular; let the stretch ratio  $\lambda = l/l_0$  be given. Then it finds after differentiation according to

the principal stresses 
$$\sigma_\alpha = p + \sum_{p=1}^N \mu_p \lambda_p^{\alpha_p} \quad (2-25)$$

with values for Neo-Hookean, Mooney-Rivlin and Ogden material. Pressure  $p$  is determined from incompressibility and boundary condition  $\sigma_2 = \sigma_3 = 0$ . With (2-20) the constitutive equation reduces to a single equation of the form

$$P = \sum_{p=1}^N (\mu_p \lambda_p^{\alpha_p-1} - \mu_p \lambda_p^{-\frac{1}{2}\alpha_p-1}) \quad (2-26)$$

For the Arruda-Boyce model we get by differentiation

$$P = \mu_0 \left( 1 + \frac{1}{5\lambda_{lock}^2} \left( \lambda^2 + \frac{2}{\lambda} \right) + \frac{11}{175\lambda_{lock}^4} \left( \lambda^2 + \frac{2}{\lambda} \right)^2 \right) (\lambda_1 - \lambda_1^{-2}) \quad (2-7)$$

The Blatz and Ko model coincides in the incompressible case with the Neo-Hookean solid. Figure 3.1 shows the corresponding stress-strain curves for rubbery materials with material data from Table 2.1.

### 2.2.2 Viscoelasticity

A viscoelastic substance loses energy when a load is applied, then removed. Hysteresis is observed in the stress-strain curve, with the area of the loop being equal to the energy lost during the loading cycle.[22] The ability to dissipate energy is one of the main reasons for using viscoelastic materials for any application to cushion shock, from running shoes to packing materials[16]. The two other main characteristics associated with viscoelastic materials are stress relaxation and creep.

In viscoelastic material energy changes to heat while exposes and releases by a load.

### ***2.2.2.1 stress relaxation***

Stress relaxation testing provides a methodology for investigating the viscoelasticity of rubber or rubberlike materials. Stress relaxation refers to the behavior of stress reaching a peak and then decreasing or *relaxing* over time under a fixed level of strain.

### ***2.2.2.1 Creep***

The long-term stress-strain behavior of polymers is generally more important than short-term properties where the product is expected to sustain a stress or strain in service. Not like most of the metals which only exhibit creep at higher temperatures, polymeric materials possess significant creep even at room temperature.

Polymers are used in applications that demand high performance and extreme reliability. Many components, conventionally made from metals, are now made from polymers. Thorough studies on the behavior of polymer under long-term load and varying temperatures are needed.

When a polymer is subjected to a constant load, it deforms quickly to a strain roughly predicted by its stress-strain modulus, and then continues to deform slowly with time-dependant response until rupture or yielding causes failure, sometimes described as the primary, secondary and tertiary phase of creep [23]. This phenomenon of deformation under constant load over time is called creep. The secondary phase is dominant and is essentially combined with the primary phase in engineering models for engineering applications. The tertiary phase is important as an indication of initial rupture [24]. At high stresses, the rupture occurs sooner than at lower stresses. However, at low enough stresses failure may never occur [24]. All polymers creep to certain extent which its degree depends upon several factors, such as amount of load, loading

time, temperature, types of plastic, the component geometry and fabrication method [24]. Figure 2.4 showed the effect of stresses on creep curves.

As mentioned previously, the long-term properties are more useful, the short-term stress-strain data is of little practical value in actual designing the part, since such data does not take into account the effect of long-term loading on elastomers. Creep behavior varies considerably among types of plastics; however, under proper stress and temperature conditions, all elastomers will exhibit a characteristic type of creep behavior.

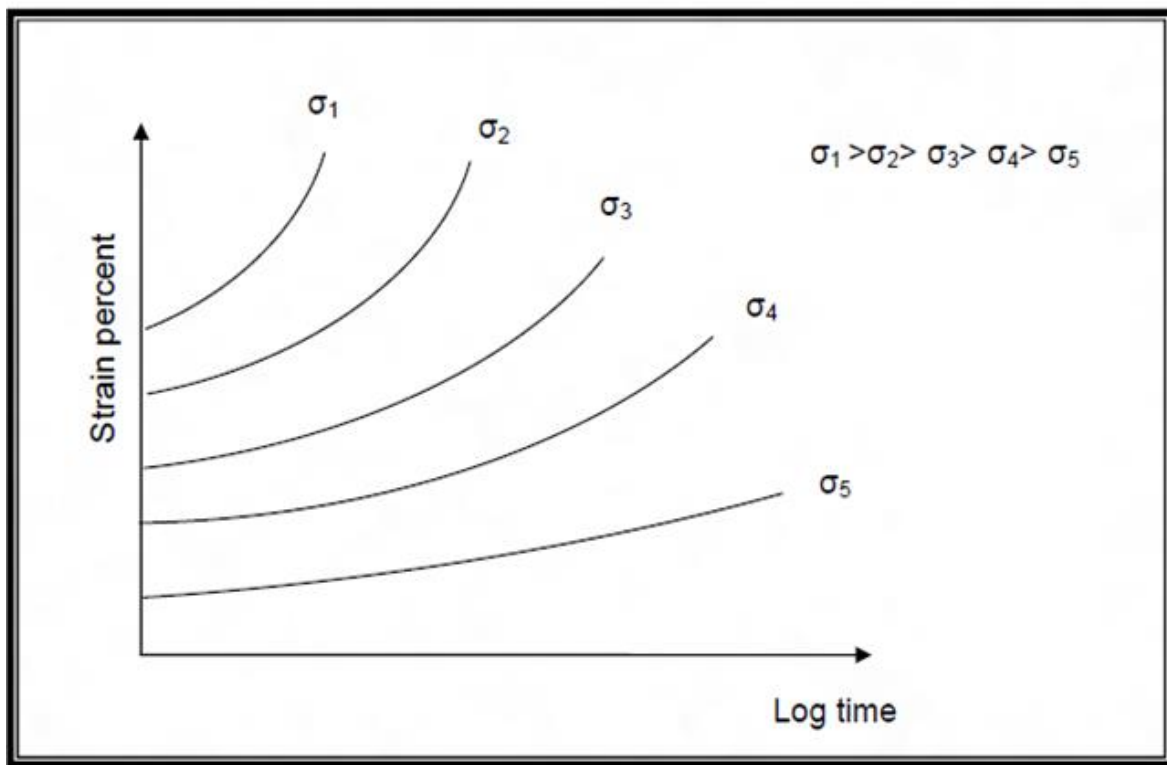


Figure 2.3: Tensile creep curve with various stresses [25, 26]

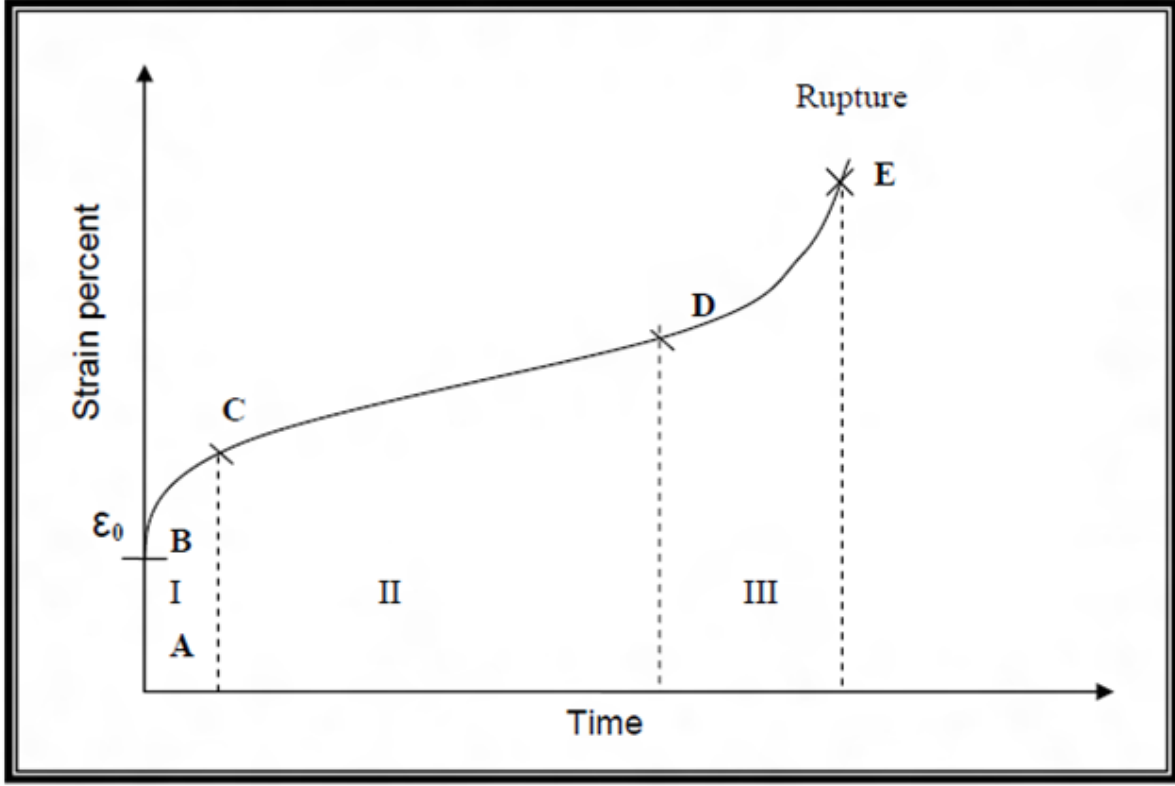


Figure 2.4: Characteristic creep Curve with linear time scale [27]

The response of the specimen loaded by  $\sigma_0$  at time  $t=0$  can be divided into an elastic and a plastic part as

$$\epsilon_0 = \frac{\sigma_0}{E(T)} + \epsilon_p(\sigma_0, T) \quad (2-28)$$

Where  $E(T)$  is the modulus of elasticity. The creep strain in Figure 2.6 can then be expressed according to

$$\epsilon_c = \epsilon(t) - \epsilon_0 \alpha t^k \quad (2-29)$$

Where  $k < 1$  in the primary,  $k=1$  in the secondary and  $k > 1$  in the tertiary creep stage. These terms correspond to a decreasing, constant and increasing strain rate, respectively, and were introduced by Betten [25] and Andrade [27].

The result of equations (2.4) and (2.5) from the creep test justified a classification of material behavior in three disciplines: elasticity, plasticity and creep mechanics.

The creep curve in Figure 2.6 shows that there is typically an almost instantaneous elastic strain AB followed by a time-dependent strain, which divided into three stages: primary or transient creep, BC (stage I), secondary or steady-state creep, CD (stage II) and tertiary or accelerated creep, DE (stage III) [28]. The initial strain, AB represents the instantaneous elastic deformation. In primary stage, the strain occurs rapidly but at a decreasing rate. The dislocation microstructure develops to reduce strain rate at this stage. It is essentially similar in mechanism to retarded elasticity and as such, is recoverable if the stress is removed.

The straight portion of the curve (CD) is characterized by a constant rate of creep. The strain rate during this state is commonly referred to as the creep rate. It determines the useful life of the material. The equilibrium of deformation and recovery mechanisms is established to maintain this constant rate of creep. This process is also called “cold flow”. It is non-recoverable because it is essentially viscous in character. The stage III is marked by an increase in creep rate as an increase in the true stress due to the cross section reduces (necking) and leading to failure or creep rupture typically logarithmic curve [23].

If the applied load is released before the creep rupture occurs, an immediate elastic recovery, substantially equal to elastic deformation followed by a period of slow recovery is observed. The material in most cases does not recover to the original shape and permanent set remains. The magnitude of the permanent set depends upon length of the time, amount of stress applied, and temperature [23].

The total mechanical response of a specimen includes perfect elasticity (related with the instantaneous strain in Figure 2.7), a coupling of elastic and viscous components, called anelasticity and a total irreversible flow [29]. The creep behavior of a polymer can be represented by an appropriate combination of mechanical models of Maxwell and Voight element. Although there are no discrete molecular structures which behave like the individual elements of the models, but they aid in understanding the response of polymer materials.

The creep test is more common and probably simpler since dead weight loading can be used with multiple creep stations. The uniaxial tensile test is considered most useful for producing accurate and consistent results that can be easily interpreted [30]. Creep data is usually obtained for a number of different stresses, as creep modulus will only independent of stress over limited ranges. It may also be important to obtain data as a function of temperature. Commonly, isochronous stress-strain curves are derived from the creep curves at different stress levels as a useful way of displaying the information.

Tensile creep measurements are made by applying the constant load to a tensile test specimen and measuring its extension as a function of time. The extension measurement can be carried out in several different ways. The simplest way is to make two gauge marks on the tensile specimen and measure the distance between the marks at specified time intervals. The percent creep strain is determined by dividing the extension by initial gauge length and multiplying by 100.

For a linear viscoelastic material, the effect of an applying stress,  $\sigma$  to the strain,  $\epsilon(t)$  can be divided into three parts by assuming linearity [31]

(I)  $\epsilon$ , an essentially instantaneous response, similar to that of an elastic solid; 1

(II)  $\epsilon(t)$ , which tends to a constant value as  $t$  tends to  $\infty$ ; and 2

(III)  $\epsilon(t)$  which is linear in time. 3

So, a time-dependent creep compliance,  $J(t)$  can be defined as [31]

$$J(t) = \frac{\epsilon(t)}{\sigma} = \frac{\epsilon_1}{\sigma} + \frac{\epsilon_2(t)}{\sigma} + \frac{\epsilon_3(t)}{\sigma} = J_1 + J_2(t) + J_3(t) \quad (2-30)$$

The creep percentage is plotted against time to obtain a tensile creep curve as illustrated in Figure 2.7. The  $J_3(t)$  corresponds to flow and will be assumed as zero. The term  $J_1$  corresponds to a response that is faster than can be observed experimentally, rather than an instantaneous one. The strain  $\epsilon_1$  can be called the unrelaxed response, in contrast to the relaxed response observed at long time  $\epsilon(\infty)$ .  $J$  and  $J_2(t)$  are not usually considered separately in what follows, so that  $J(t)$  is equal to  $J+J(t)$  [31]



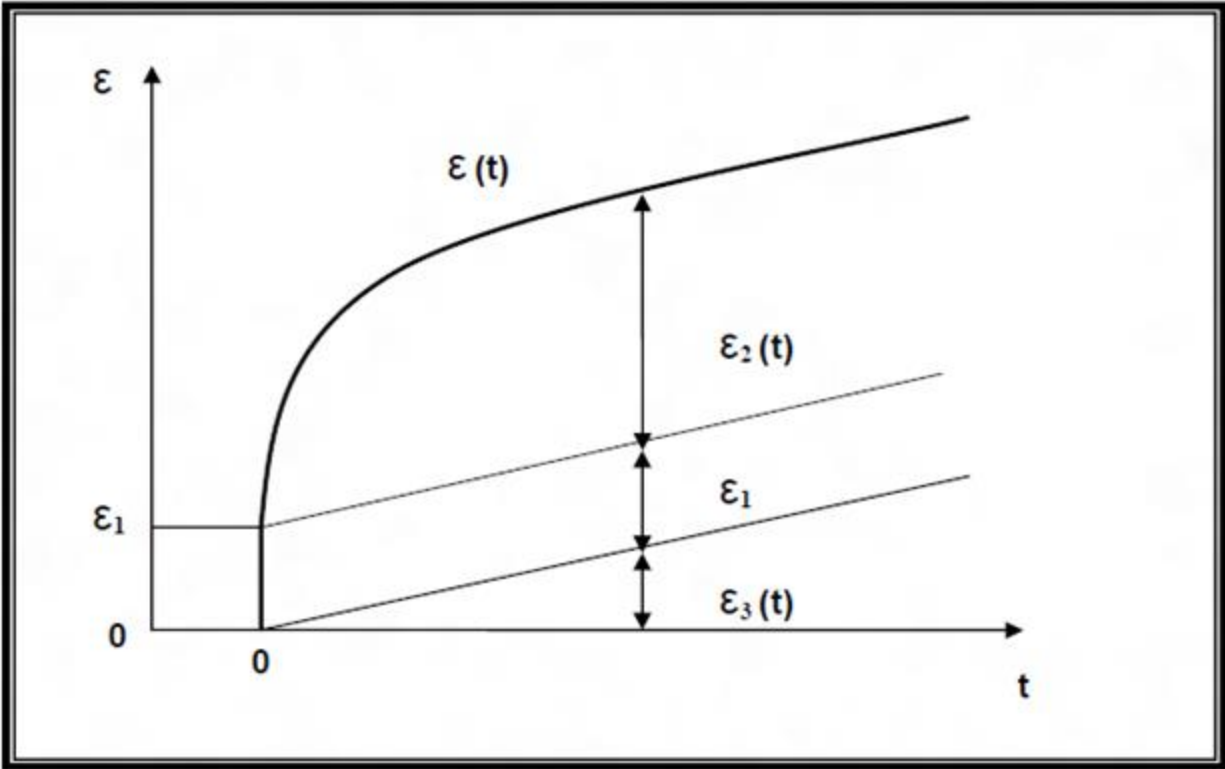


Figure 2.5: Creep of a viscoelastic under a constant stress [31]

The tensile stress values are also determined at specified time intervals to facilitate plotting a stress-rupture curve. The more accurate measurements require the use of a strain gauge, which is capable of measuring and amplifying small changes in length with time and directly plotting them on a chart paper. The test is also carried out at different stress levels and temperatures to study their effects on tensile properties.

### ***2.2.3 Swollen Elastomers***

All long, flexible polymer molecules naturally become entangled, like spaghetti. Although all such molecules will disentangle and flow under stress, their physical entanglements will act as temporary “interlinks,” especially when the molecules are long and slow-moving.[32] It is therefore difficult at first sight to distinguish a covalently interlinked elastomer from one that is merely tangled or (as is described below) one that is held together by strong intermolecular associations. One means of distinguishing is to test whether the polymer dissolves in a compatible solvent or merely swells without dissolving [33, 34]. Covalently interlinked molecules do not dissolve. Interlinking is therefore necessary for good solvent resistance or for use at high temperatures.

Crosslinked elastomers networks can absorb solvents and swell many times to their initial volume. Swelling is used by plants to regulate the transport of water,(1) and is exploited in consumer products such as contact lenses (2) and superabsorbent diapers. (3) Swelling elastomers have been developed as vehicles for drug delivery, (4) and as actuators and sensors in microfluidics. (5) [24]

## **2.3 Finite Element Analysis**

### ***2.3.1 Terminology***

In order to understand FEA, several terms need to be defined and understood:

1. Element. The structure to be analyzed is broken down into pieces called "elements," which are usually triangles or quadrilaterals. When these elements are joined together they form a mesh which fully describes the geometry of the component to be analyzed.

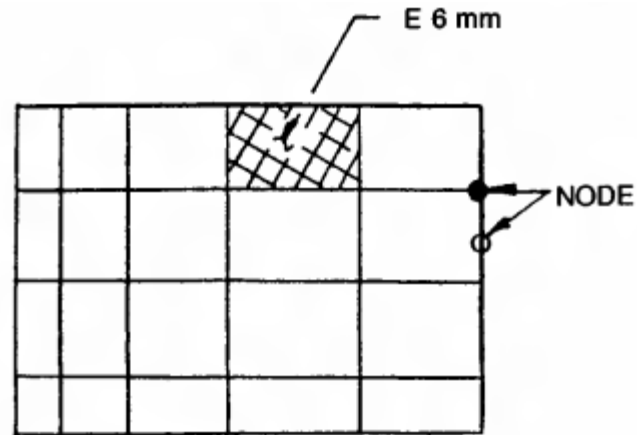
2. Node. A "node" is a point where elements are joined. There is always a node at the corners of the elements, and some elements also have mid-side nodes, which need to be specified. A mid-side node adjacent to an element without a midside node needs to be tied back to the corner node.

3. DOF. Each node has certain degrees of freedom (DOF). This means that the node is capable of moving in various directions, depending on the boundary conditions imposed on the node. The possible movements consist of displacements in three mutually perpendicular directions and rotation about these three axes for a total of six DOF.

4. Mesh or Grid. These two terms are generally used interchangeably and refer to the joined elements that look like a "grid" or "mesh".

5. Boundary Conditions. The "boundary conditions" describe the loading to be applied to the component, and include deflections, pressures, forces, body forces, friction, and thermal loads.

6. Contact Surfaces. Contact surfaces are like boundary conditions, but they do not need to be in initial contact in the FEA model. It may be specified that the component can stick, can slide but not lift off; or can lift off, but not slide on the contact surface. Contact surfaces are rigid bodies. A FEA program for the analysis of elastomer components needs a robust contact surface capability [35]



(a) MESH OF GRID



(b) DEGREES OF FREEDOM

Figure 2.6: FEA Nomenclature

### 2.3.2 Types of FEA Models

Four types of FEA models can be constructed depending on the component to be analyzed and the capabilities of the FEA program being used. The most common model is a two-dimensional (2-D) cross-section of the component with an axis of symmetry. This is also called an axis-symmetric model. The second type of model is a plane strain model, which is also a cross-section of the component. This model does not necessarily have a plane of symmetry. A generalized plane strain problem assumes that the out-of-plane displacement is constant over the entire model.

The third type of model is a plane stress model, which is like the plane strain model except it is assumed that the out-of-plane stress is constant. This model has some thickness to it. If you had a long strip of seal and wanted to evaluate how it would react to a uniform compression deflection, then a plane stress model could be used.

The fourth type of FEA model, and the most complex, is a three-dimensional (3-D) model. This type of model does not usually have a plane of symmetry or a continuous cross-section. A three-dimensional model is generally difficult to build and requires skill in interpreting the results. It should only be attempted after experience in using 2-D models. Sometimes a three dimensional model does not need to be a full 3-D model if a plane of symmetry is present, or if the component is cyclic symmetric. It is important to keep the number of elements to a minimum in a three-dimensional model.

### ***2.3.3 Model Building***

The initial step in solving a problem using FEA is to take the geometry of the component and break it down into an element mesh that accurately describes the geometry. This is accomplished by locating each node in a defined coordinate system and connecting the nodes to form the elements. During this connectivity operation, the element type and the element material type are usually specified. The element type needs to be compatible with the type of model, the material properties and the type of analysis (linear or non-linear). Care must be taken to provide a sufficiently fine grid in regions that have high strain gradients[21].

These regions may be determined by experience, or by solving the FEA problem using an initial coarse mesh to locate the high gradient regions and refining the element mesh in these regions.

Another method to obtain a finer mesh in local areas is to invoke "adaptive meshing" which is available in some FEA programs. This automatically refines the mesh as specified criteria, such as strain energy level or elements in contact, are encountered. These regions usually occur at discontinuities such as fillets, corners, at points of contact, and at the edge of the elastomer near a bond line.

Every attempt should be made to construct the FEA model using quadrilaterals, avoiding triangular elements. Triangular elements are generally stiffer than quadrilaterals and usually exhibit constant strain across the element. It is true that if sufficient triangular elements are specified, the solution approaches the quadrilateral solution. The penalty is too many elements and high run times. Some programs on the market today scan the elements and identify those which have a poor aspect ratio, or are skewed too much. If triangular or poor quadrilaterals are unavoidable, locate them in non-critical regions of the model.

The elastomer portion of the component needs to have an element specified which is formulated to handle the near incompressibility of rubber. This is generally accomplished using a Herrmann "mixed method" solution for incompressible and nearly incompressible isotropic materials. His solution incorporates a "mean pressure" function as an independent variable. These elements need to be compatible with the elements specified for the non-elastomeric components.

Some FEA programs permit the geometry to be defined within the pre-processor; some require the geometry to be defined in some type of CAD package; and some permit both methods to be used. Most FEA programs read, as a minimum, an IGS file, while some have translators for most popular CAD programs. It is generally significantly quicker to define the geometry in a CAD program and read an IGS file into the FEA program for meshing than to attempt to define the

geometry within the FEA package, which is normally not intended to be used as a full CAD package[21].

#### ***2.3.4 Modeling Hints For Non-Linear FEA***

1. In non-linear FEA, you usually need a more refined model than in linear FEA. In other words, a given mesh is less accurate in predicting non-linear response than in predicting linear response, except in very special cases.
2. If the mesh distorts badly during a non-linear analysis, you need to re-zone the model and change the mesh density in the deformed shape between load steps.
3. Keep the model as simple as possible.
4. A 3-D problem is always more complicated to analyze than a 2-D one. This is especially true in non-linear FEA. You should do your best to represent the problem in two dimensions, and see whether it makes sense to solve it as a plane strain, plane stress, or axi-symmetric problem. A simple model is much easier to develop, validate, execute, and evaluate.
5. Whenever possible, take advantage of symmetry in the structure and loading to reduce the size of your model. This is particularly true in non-linear FEA.
6. In non-linear FEA, lower-order elements are often preferred over higher-order elements, because of reasonable accuracy at reduced cost and their robustness for large deformation analysis and contact problems. Use a linear element in preference to quadratic and cubic elements.

7. When using lower-order elements, 4-node quadrilaterals are generally preferred over 3-node triangles in 2-D problems. Similarly, 8-node brick elements perform significantly better than 4-node tetrahedra in 3-D problems. In non-linear FEA, such as plasticity and rubber analysis, it is well known that the "linear" 3-node triangle and 4-node tetrahedron can give incorrect results because they are too stiff. The implication is that if you are using a pre-processor that generates such triangular and tetrahedral elements for a non-linear analysis, be careful of the results.
8. If you are using elements with different degrees of freedom, you need to provide appropriate constraints to account for the dissimilarity.
9. When constructing the mesh, place elements so that discontinuities in loads and material properties are located on the element boundaries, not inside elements. Three common discontinuities are (1) interfaces between different material or physical properties; (2) step changes in loads; (3) concentrated loads.
10. Be careful how a joint is modeled. Is it stiff, does it have some rotation, or can it be smeared?
11. Be careful of how the support structure is modeled. If the stress-strain data around the support point are of interest, then there needs to be mesh refinement in that area and the support needs to be defined accurately.



### ***2.3.5 Boundary Conditions***

After the mesh is generated, the boundary conditions are applied. For all types of FEA analysis, the boundary conditions need to be applied with caution. FEA of Elastomers requires extreme caution due to the special nature of elastomeric materials.

The most common boundary condition is when a surface of the component is firmly attached to another component or the ground. When the FEA surface (or node) is fixed, all of the DOFs are set equal to zero and the node cannot move in any direction. If the component is resting on a surface, but can move parallel to it, then only the DOF normal to the surface needs to be set to zero, leaving the component free to move parallel to the surface. This is like setting the surface on rollers.

The loading can be applied as a force or pressure. Forces are usually applied at the nodes, while a pressure is applied along a surface. In either event, it is essential to remember the type of model being used and specify the loading accordingly. A plane problem usually requires the loading to take into consideration the length and depth of the model. In an axi-symmetric problem, the modeled cross-section is assumed to be a part of a 360° model. Therefore, the loading is either specified per radian, per arc length, or as force per unit area (pressure).

The loading can be applied as a deflection at the nodes, along a surface, or with a rigid body contact surface. If you know how a surface moves, this is the easiest and most appropriate method of loading an elastomeric component. To apply a force or a pressure over a surface, information about how it is distributed (which is usually unknown) is necessary.

Application of a pressure to an elastomer surface for a non-linear analysis introduces potential problems. The pressure has to be applied in steps to arrive at a solution. While this iterative

procedure is attempting to arrive at an equilibrium point for each load step, significant changes in geometry may occur. This requires very small pressure increases in each step to attain convergence, and the pressure needs to be specified as following the surface to which it is applied. The use of contact surfaces is an excellent way to apply a deflection to a model.[21] The characteristics of the contact surface can be specified to let the component stick, lift off, or slide on the surface. Friction can also be specified.

### ***2.3.6 Solution***

Modern non-linear FEA programs offer "automated" load stepping procedures to help users find the best solution at the least cost. These solution techniques should be more accurately called "semi-automatic," because the engineer still needs to make decisions regarding the tolerance desired in the answers, convergence criteria, load/time step size selection, appropriateness of material properties, the need for adaptive meshing, mesh rezoning, uniqueness or instability of the solutions, etc.

#### ***2.3.6.1 Tangent Stiffness***

In large deformation analysis, the relationship between incremental load and displacement is called a tangent stiffness. This stiffness has three components: the elastic stiffness, the initial stress stiffness and the geometric stiffness. The elastic stiffness is the same as that used in linear FEA. The second term represents the resistance to load caused by realigning the current internal stresses when displacements occur. The third term represents the additional stiffness due to the non-linear strain-displacement relationship.

In solving this type of problem, the load is increased in small increments, the incremental displacements are found, the next value of the tangent stiffness is found, and so on. There are three approaches available to solve these types of problems:

1. Total Lagrangian Method refers everything to the original undeformed geometry. This is applicable to problems exhibiting large deflections and large rotations, but with small strains, such as thermal stress, creep, and civil engineering structures. It is also used in rubber analysis where large elastic strains are possible.
2. In the Updated Lagrangian Method, the mesh coordinates are updated after each increment. This applies to problems featuring large inelastic strains such as metal forming.
3. In the Eulerian Method, the mesh is fixed in space and the material flows through the mesh. This is suitable for steady-state problems, such as extrusion and fluid mechanics problems.

#### ***2.3.6.2 Newton-Raphson***

Two popular incremental methods used to solve non-linear equilibrium equations are full Newton-Raphson and modified Newton-Raphson. The full Newton-Raphson (N-R) method assembles and solves the stiffness matrix at every iteration, and is thus expensive for large 3-D problems. It has quadratic convergence problems, which means that in subsequent iterations, the relative error decreases quadratically. It gives good results for most non-linear problems. The modified Newton-Raphson method does not reassemble the stiffness matrix during iteration. It costs less per iteration, but the number of iterations may increase substantially over that of the full N-R method. It is effective for mildly non-linear problems.

### ***2.3.6.3 Non-Linear Material Behavior***

When stresses go beyond the linear elastic range, material behavior can be broadly divided into two classes: (1) time-independent behavior (plasticity that is applicable to most ductile metals; non-linear elasticity that is applicable to rubber); (2) time-dependent behavior (creep, and viscoelasticity that are applicable to high-temperature uses; viscoelasticity that is applicable to elastomers and plastics).

Creep is continued deformation under constant load, and is a type of time-dependent inelastic behavior that can occur at any stress level. Creep is generally represented by a Maxwell model, which consists of a spring and a viscous dashpot in series. For materials that undergo creep, with the passing of time, the load decreases for a constant deformation. This phenomenon is termed relaxation.

### ***2.3.6.4 Visco-Elasticity***

Visco-elasticity, as its name implies, is a generalization of elasticity and viscosity. It is often represented by a Kelvin model, which assumes a spring and dashpot in parallel. Rubber exhibits a rate-dependent behavior and can be modeled as a visco-elastic material, with its properties depending on both temperature and time (creep, stress relaxation, hysteresis). Linear visco-elasticity refers to a material which follows the linear superposition principle, where the relaxation rate is proportional to the instantaneous stress. It is applicable at small strains. Non-linear visco-elasticity behavior may result when the strain is large. In practice, modified forms of the Mooney-Rivlin, Ogden, and other polynomial strain energy functions are implemented in non-linear FEA codes.

### ***2.3.6.5 Model Verification***

After the mesh is generated, it should be plotted to verify

1. that the mesh is defining the component correctly
2. that the nodes are correctly joined
3. that the correct boundary conditions are applied at the correct locations
4. that the correct material property is specified for each element
5. that the correct element types are specified at the correct locations

### ***2.3.7 Results***

Normally, data extracted from FEA solutions for steel or aluminum components are in terms of stress because the stress levels of one steel part can be readily compared with another steel part, and most of the material property data is expressed in terms of stress. On the other hand, comparison and evaluation of FEA stress results from the analysis of elastomeric components can be made only if the specified elastomer material properties are identical. Comparing the analysis of a component using a shear modulus  $G = 1 \text{ MPa}$  (145 psi) with another using  $G = 1.5 \text{ MPa}$  (217.5 psi) is not valid in terms of stress.

Evaluation and comparison of elastomeric component analytical results need to be done in terms of strain energy if different elastomers are to be considered. If the component with  $G = 1 \text{ MPa}$  has a shear stress of  $0.5 \text{ MN/m}^2$ , its shear strain is 50% strain, and its strain energy is  $125 \text{ kJ/m}^3$ , while the component with  $G = 1.5 \text{ MPa}$  under the same shear stress would have a shear strain of 33% and a shear strain energy of only  $83 \text{ kJ/m}^3$ .

The interface between the elastomer and the metal to which it is bonded is a critical location to be checked during the analysis. A significant number of finite element programs do not distinguish between the two materials when the data along this interface is processed. The stress levels in the steel and the stress levels in the elastomer are averaged at the nodes and reported. This is not valid. When carrying out FEA of an elastomer component with metal as part of your model, verify that the program is in fact reporting the actual stress-strain values in the elastomer and not "smearing" the data from the metal together with the elastomer results. Various forms of plotted output can usually be obtained from FEA results.

The movement of a particular point within a component, or the change in stress or strain during a non-linear analysis, is sometimes of particular interest. This may be done using a history plot. The better the plotting capability of the FEA program, the more usable is the analysis.

## ***CHAPTER 3: RESEARCH METHODOLOGY***

Dynamic stress analysis models for rubber must include time dependent material behavior to be realistic. For some materials, step-strain relaxation test data suggests that the stress can be approximated as a linear superposition of time independent (long term) and the time dependent (short term) components. For example, carbon black filled rubber-like materials often have short term stress components equal to or larger in magnitude than their long term or quasi-static components[30]. This is especially true at large strains in which case the time dependent components are known to depend on the strain state for step-strain tensile, biaxial and shear tests. Methods for analyzing large dynamic strains and stresses in rubber have been under development for at least forty years and a great deal of insight and understanding has been obtained from those studies. Typical issues of interest to engineers are: (a) Can the viscoelastic model be used to predict material response for loadings other than those used to determine its constants, and (b) can its constitutive constants be determined from tests which are not difficult to conduct?

For example, quasi-static constitutive models for rubber are typically determined by fitting the stress data obtained from tensile, biaxial and shear tests to constitutive formulas computed from assumed forms of a strain energy density function.

### ***3.1 Materials***

The data related to the materials used in present study are obtained from the experiments conducted by other students in our department [32]. Commercial grade of NBR with 60 shore hardness  $A \pm 5$  used in this research were provided by MAKKA Engineering Sdn. Bhd., Malaysia. The NBR has specific gravity of  $1.4 \pm 0.1$  and 25 wt.% of carbon black. For this type of rubber

compound, the vulcanization process was performed by compression molding process at 165 °C for 5 min under a pressure of approximately 6.89 MPa from an electrical resistance heating press. The rubber specimens for swelling and mechanical tests are annular cylindrical block with outside diameter of 50 mm, inner diameter of 38 mm and height of 10 mm. Note that no standard is followed in the determination of specimen geometry. Indeed, the wall thickness of the specimen is chosen such that swelling in the specimen during the immersion test can occur in a relatively short period of time while ensuring that specimen buckling will not occur during the compression test.

The palm biodiesel (B100) was purchased from Am Bio-fuels Sdn. Bhd., Malaysia. Table 3.1 shows the analysis report of the palm biodiesel used in the present study.

Test	Unit	Methods	Results
Ester content	% (m/m)	EN 14103	96.9
Density at 15 °C	kg/m <sup>3</sup>	EN ISO 12185	875.9
Viscosity at 40 °C	mm <sup>2</sup> /s	EN ISO 3104	4.667
Flash point	°C	EN ISO 3679	168
Cetane number	–	EN ISO 5165	69.7
Water content	mg/kg	EN ISO 12937	155
Acid value	mgKOH/g	EN ISO 3679	0.38
Methanol content	% (m/m)	EN 14110	<0.01
Monoglyceride content	% (m/m)	EN 14105	0.67
Diglyceride content	% (m/m)	EN 14105	0.2
Triglyceride content	% (m/m)	EN 14105	0.2
Total glycerine	% (m/m)	EN 14105	0.25

**Table 3 . 1:** Properties of B100 palm biodiesel [32]

The swollen specimens are obtained by immersion duration the dry specimens in biodiesel for two and five days.



### 3.2 Elasticity

Let us consider an infinitesimal material neighborhood undergoing a deformation along a path “ $\Gamma$ ”. The deformation is defined by a deformation gradient  $F : [t_1, t_2] \rightarrow GL+(3, \mathbb{R})$ .

Then, the work of deformation associated with this path is:

$$W = \int_{t_1}^{t_2} P(t) \cdot F(t) dt \quad (3-1)$$

A material is said to be an elastic material if the work of deformation is path independent .  
Consequently holds:

$$\int_{\Gamma'} P_{ij} dF_{ij} = \int_{\Gamma''} P_{ij} dF_{ij} \quad (3-2)$$

for all paths of deformation  $\Gamma', \Gamma'' \in GL+(3, \mathbb{R})$  defined by functions  $F', F'' :$

$[t_1, t_2] \rightarrow GL+(3, \mathbb{R})$  such that  $F'(t_1) = F''(t_1)$  and  $F'(t_2) = F''(t_2)$ .

For all closed paths of deformation the work of deformation is zero.

#### 3.2.1 Variational form

The definition of elasticity implies that for any deformation path  $\Gamma$  starting at a fixed reference placement and terminating at  $F$  the strain energy density  $W$  is of the form :

$$W = \int_{\Gamma} P dF \quad (3-3)$$

Clearly, the elastic strain energy density is a function of the deformation only. In addition we know the gradient, i.e., the work conjugate stress tensor:

$$P_{ij} (F) = \frac{\partial W}{\partial F_{ij}} (F) \quad (3-4)$$

The strain energy density acts as a potential for the stress tensor .Relation (3-4) is, therefore, the general variational form of elastic constitutive laws[30]. Elastic materials with variational constitutive relations like (3-4) are also called hyperelastic materials. In contrast to that, models with an ad hoc formulation of the elastic constitutive law are called hypoelastic materials. The hypoelastic constitutive relation is formulated in rate form, i.e., the stress rate is defined. In original hypoelastic theory [53], the stress rate is a function of the rate of deformation tensor (2.47) and additional contributions, e.g., the stress itself  $f(d_{kL}, \sigma_{ij}, \dots)$ .

However, such constitutive relations (which are not elastic in the sense of the above definition) are not employed in modern constitutive theories. Instead, the name hypoelastic mostly refers to a rate formulation of the elastic law, e.g.,

$$\hat{\sigma}(F) = {}^{<4>} C(F)d(F) \quad (3-5)$$

where  $\hat{\sigma}$  denotes a physical meaningful (objective) time derivative of the Cauchy stress tensor. The components of the stiffness tensor  ${}^{<4>} C(F)$  are expressions of the elastic constants which in turn depend on the actual definition of the stress rate (and thus on the deformation). Because of the properties of the rate of deformation tensor hypoelastic constitutive laws do not strictly reflect the path independence of elasticity. Moreover, the derivation of objective rates of stress tensors and the corresponding stiffness tensors is not trivial, see, e.g. [8]

From the theoretical point of view there is no reason to work with hypoelastic constitutive relations. However, the majority of commercial finite element programs still applies constitutive

relations like (3-5). For that reason we mention this approach here. In the remaining of this text we speak about elasticity meaning the constitutive equations in its variational form (3-3, 3-4).

### 3.2.2 Internal energy, stresses and elasticity tensor

The of a strain-energy function  $W(F)$  of an elastic material corresponds to the Helmholtz free energy density

To be more precise, inserting (3-4) into the mechanical energy balance equation (3.3) gives the identity :

$$U = W(F) = A(F) \quad (3-6)$$

which states that the internal energy density of an elastic body coincides (up to an inconsequential additive constant) with the strain energy density.

From the definition of elasticity it follows that a material is elastic if and only if for all closed paths of deformation the rate of free energy vanishes, i.e., if the deformation of the material does not entail dissipation or hysteresis. This yields to the definition of elasticity in terms of continuum thermodynamics where a material is said to be elastic if it produces no entropy. The second law of thermodynamics degenerates to an equation.

Following a strategy known as Coleman-Noll procedure we expand the Clausius-Planck inequality to write:

$$P \cdot \dot{F} - \dot{A} = \left( P - \frac{\partial A}{\partial F} \right) \cdot \dot{F} = 0 \quad (3-7)$$

Only if the term in brackets vanishes equation (3-7) holds for every rate of deformation. In consequence, this relates the stresses to the energy function as in the constitutive relation (3-4).

In order to obtain numerical solutions of nonlinear finite-deformation problems the linearized stress state is of central importance. Therefore we proceed expressing relation (3-4) in incremental form. This can be accomplished in a number of mathematically equivalent ways. For instance, taking differentials of (3-4) gives

$$dP_{ij} = C_{ijkl}(F)dF_{kl} \quad (3-8)$$

where  $C_{ijkl}(F)$  are the Lagrangian elastic moduli. The elastic moduli are the components of the fourth-order elasticity tensor  ${}^{<4>}C(F)$  in material description

$${}^{<4>}C = C_{ijkl} e_i \otimes e_j \otimes e_k \otimes e_L \quad \text{with} \quad C_{ijkl}(F) = \frac{\partial^2 W}{\partial F_{ij} \partial F_{kl}}(F) \quad (3-9)$$

The elasticity tensor is always symmetric in its first and second and in its third and fourth index. This symmetry is known as minor symmetry,

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{jilk} \quad (3-10)$$

If derived from a scalar-valued energy function as presumed here by (3-9), the tensor  ${}^{<4>}C$  also possesses major symmetry, i.e., it is symmetric in the sense

$$C_{ijkl} = C_{klij} \quad \Longleftrightarrow \quad {}^{<4>}C = {}^{<4>}C^T \quad (3-11)$$

A standard exercise shows that a fourth-order tensor with major and minor symmetry has only 21 independent components.

### **3.3 NBR Pipe Modeling in Ansys**

ANSYS is a very effective FEA tool for composite beams. Apart from structural analysis it can handle modal analysis, too. In the present research work, ANSYS has been used to model a pipe using material and geometries used in [1]. The pipe made of Nitrile Butadiene Rubber is a special viscoelastic material .

The basic steps of ANSYS modeling are Preferencing, Preprocessing, Solving, and Post processing and are discussed in detail in the following sections.

#### ***3.3.1 Preferences***

First step in ansys is defining the CUI filtering. There are some individual segments consist of Structural, Thermal, Ansys fluid, Flotran CFD and Electromagnetic. In this simulation Structural has chosen.

#### ***3.3.2 Preprocessor***

In this step the material properties are defined and the geometrical model is created.

##### ***3.3.2.1 Defining material properties:***

Nonlinear viscoelastic material properties; method is to first obtain the Prony parameters through a curve-fit procedure, then save all those values. ANSYS can generate the parameters and define the properties of the nonlinear viscoelastic material. Here, this method has been adopted to define the properties of the viscoelastic material.

### 3.3.2.2 The steps for Viscoelastic Curve fitting are:

Prepare Experimental Data, Input the Data into ANSYS, Select a Material Model Option, Initialize the Coefficients, Specify Control parameters and Solve, and Plot the Experimental Data and Analyze, and Write Data to TB Command.

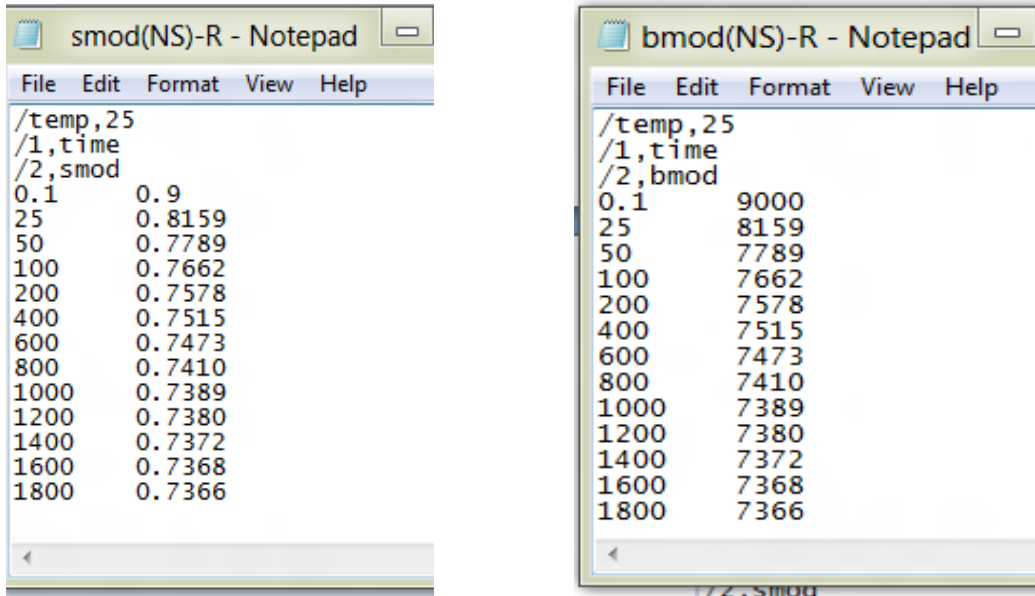


Figure 3. 1: Excerpt of data (a) time vs. shear modulus and (b) time vs. bulk modulus [5]

Preparing experimental data: in this step, time, shear stress, and/or bulk modulus data at certain temperature are /temp, 25 /1, time /2, smod or /2, bmod. Figure 3.1 shows shear modulus and Bulk modulus vs. time data prepared separately for each temperature. Here, the temperature is 25°C The data file must be a plain text file and must contain the extension “.exp.”

Enter the “case name” of the material. Here, it is named NS1. Then, consider five shears and bulk terms (expansion of first five terms of shear and bulk modulus). For temperature dependent test data, we can consider “Williams-Landau-Ferry (WLF)” or “Tool-Narayanaswamy (TN)” shift functions. First, uncheck the “partial solve” for the “bulk terms” and the “shift function” and only solve for shear terms, as shown in Figure 3.2 & 3.3. Enter the number of iterations, here

1000. Once this set of data is generated, we can repeat the same procedure for the bulk terms. Finally, check all “partial solve” box to obtain the complete solution, including the shift function, as shown in Figure 3.4. Where, we can see that all coefficients have been generated, including the WLF coefficients, C1 and C2.

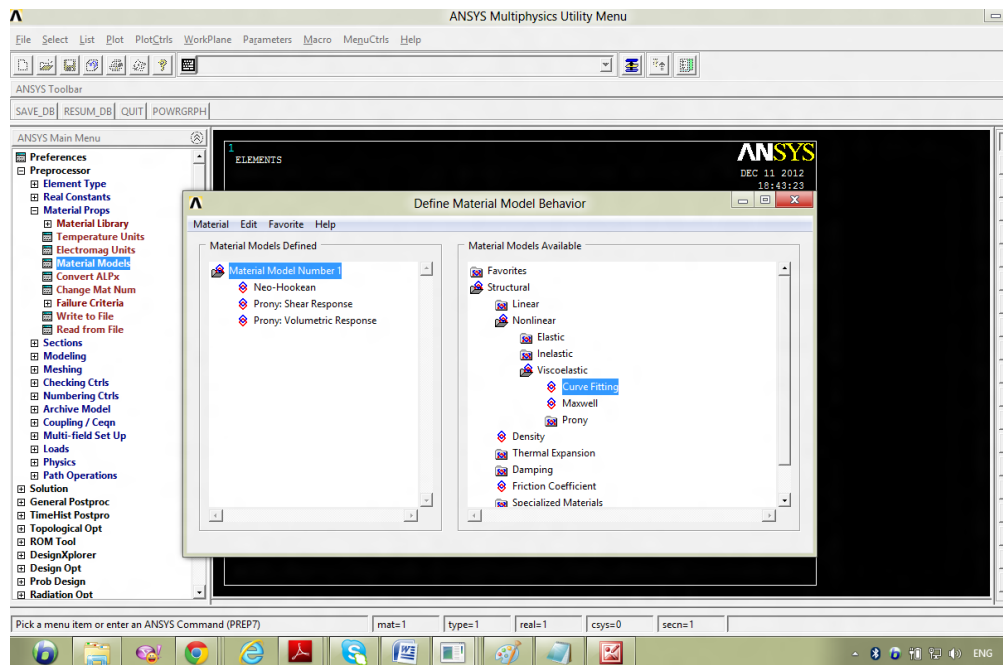


Figure 3.2: Shear terms solution data in ANSYS’s Curve fit

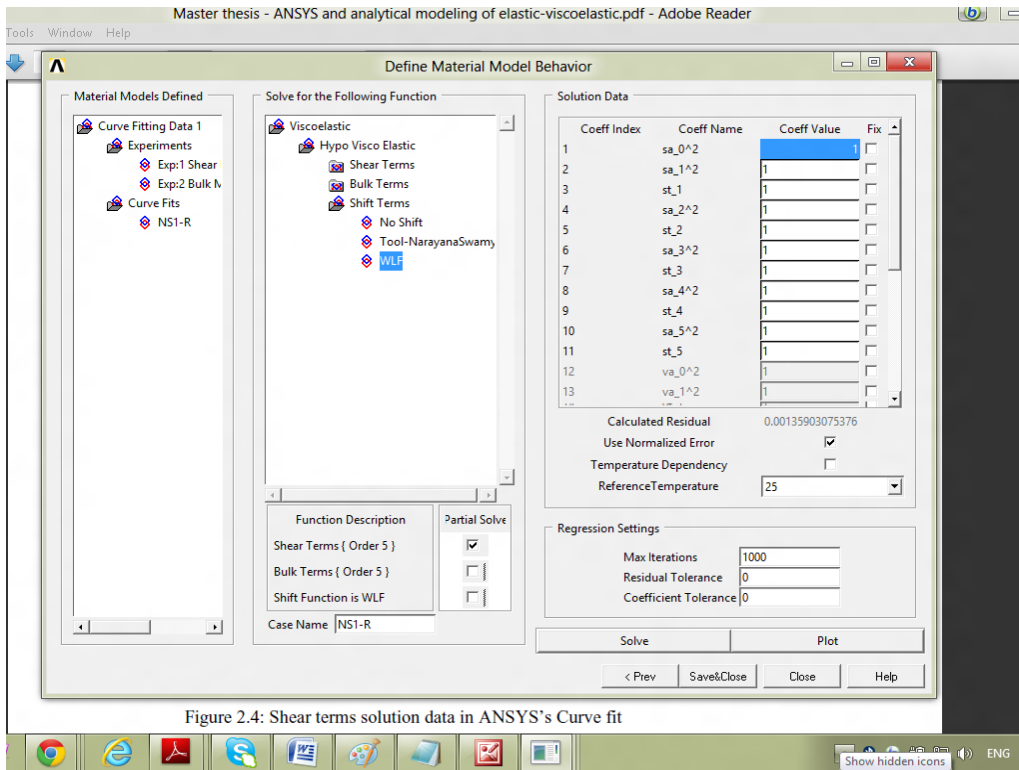


Figure 2.4: Shear terms solution data in ANSYS's Curve fit

Figure 3.3: Shear terms solution data in ANSYS's Curve fit

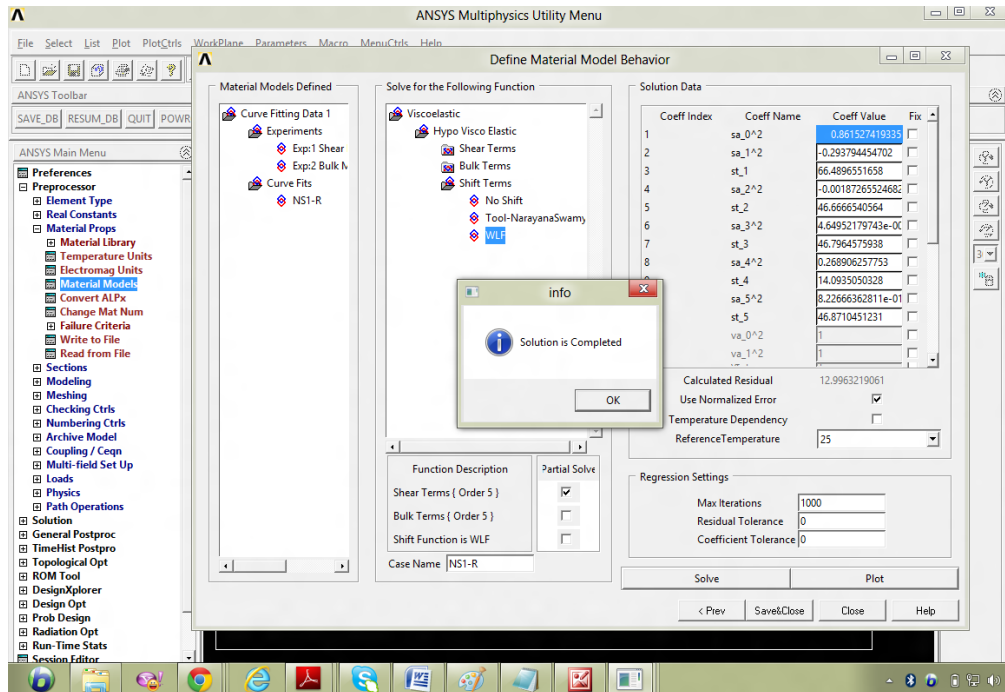


Figure 3.4: ANSYS's Curve fit generating the complete solution.



Once, we have generated the complete solution we can check its curve fit by plotting both shear terms and bulk terms vs. time, as shown in Figure 3.5. We can also compare or check any point by locating its coordinate or zoom-in on a portion of the curve. In these plots the Experimental data plot (black points) is in perfect fit with the ANSYS generated data plot (red curve) for both Shear and Bulk moduli.

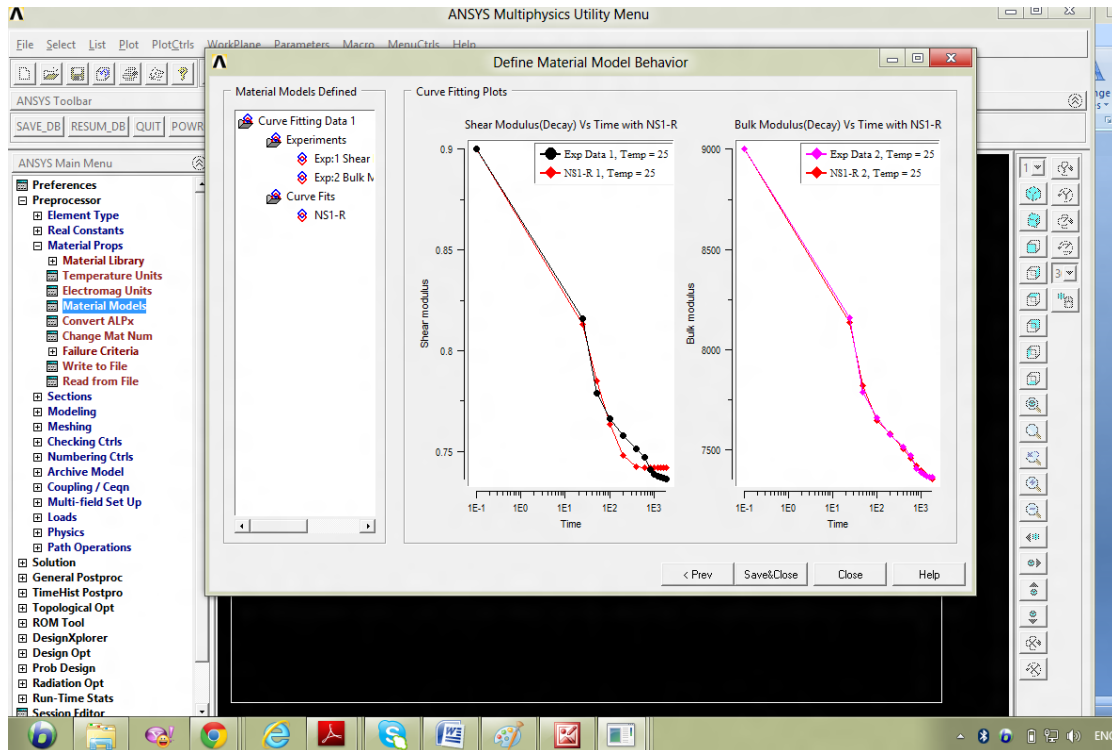


Figure 3.5: Curve fit plots for shear modulus vs. time and bulk modulus vs. time

ANSYS can generate Prony series terms by itself when we save and close the curve-fit generated results. We can check these Prony terms, as shown in Figure 3.6, when we go into Structural > NonLinear > viscoelastic > prony. We can find the shear response, bulk response and shift function, corresponding to the reference temperature. The data are shown here for the shear response only.

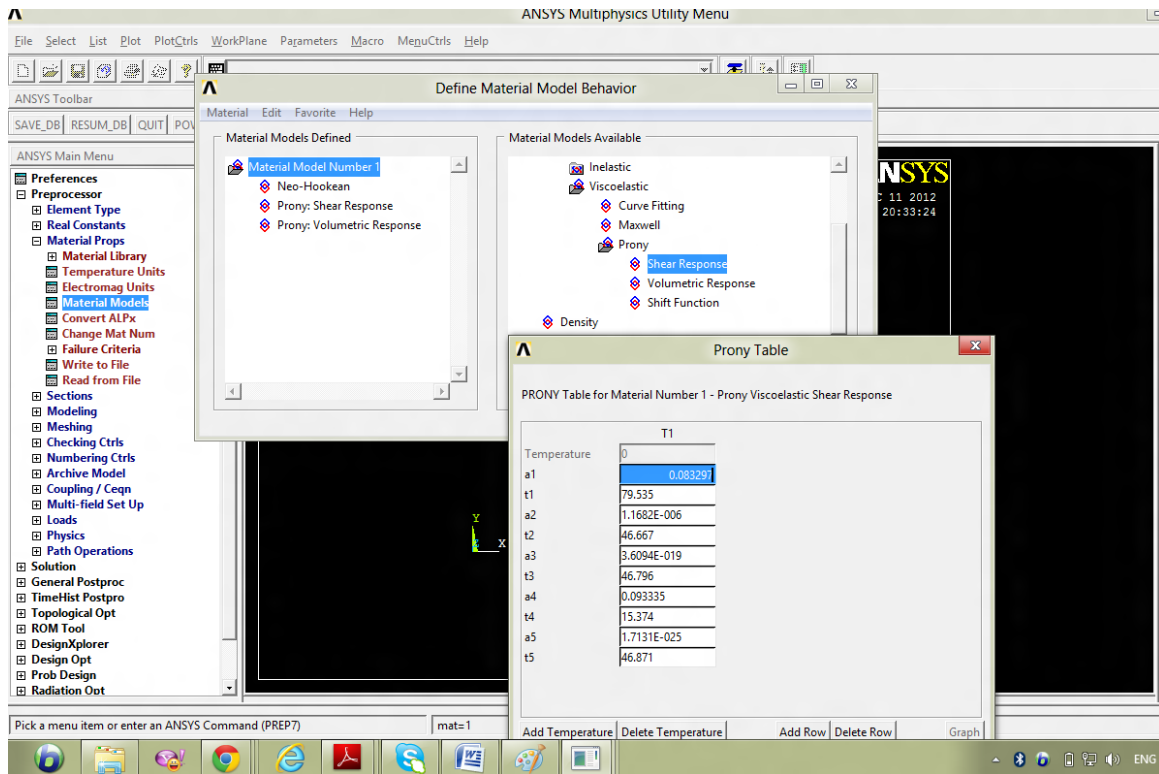


Figure 3.6: ANSYS generates the Prony terms

By the end of this step we have obtained the Prony series parameters, which mean that we have defined the nonlinear behavior as a nonlinear decrement of the shear/bulk moduli with respect to time. This completes the material defining procedure in ANSYS.

### 3.3.2.3 Geometric modeling of the Pipe:

The cylindrical shape pipe has three different dimensions L, R1 and R2 for internal and external radius, respectively. This pipe is considered with different boundary

### 3.3.2.4 Defining Pipe dimensions:

For pipe , L=150 mm, R1= 25 mm, and R2= 30 mm, these variables can be entered as “Parameters.” In some symmetric case like cylindrical shape, A quarter of shape should be

chosen to save the time and the energy. Figure 3.7 shows a partial cylinder created through dimensions.

Preprocessor > Modeling > Create > Volumes > Cylinder > Partial Cylinder

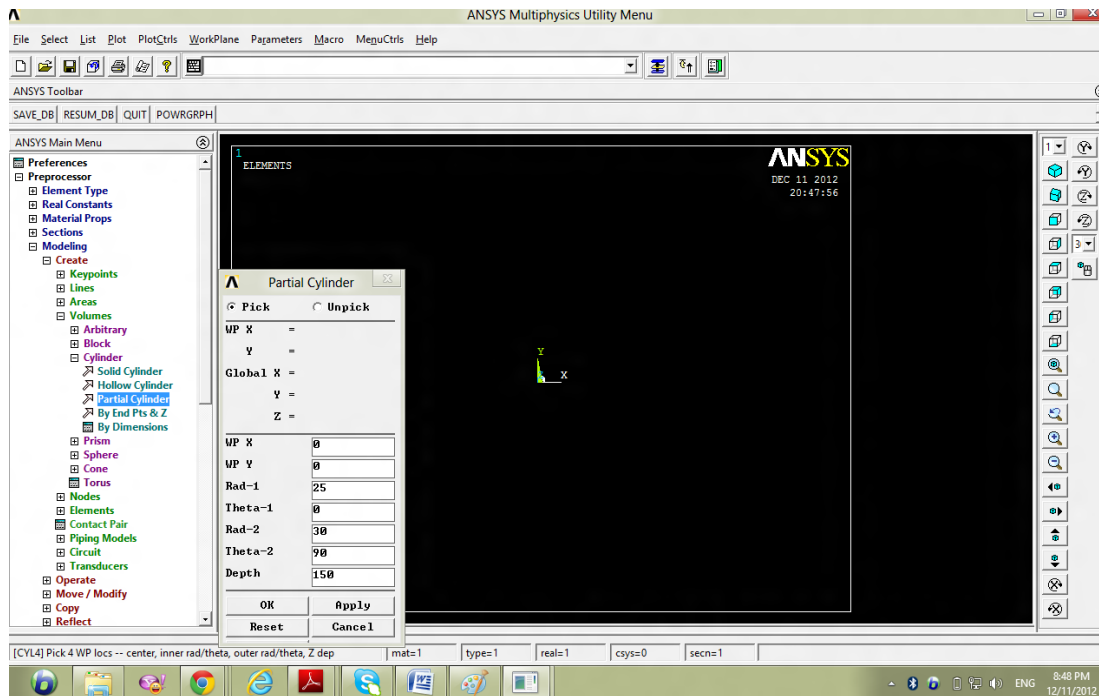


Figure 3.7: Creating quarter of Cylinder in ANSYS

### 3.3.2.5 Defining element type:

It is essential to define element type before meshing any area. Here, we are dealing with a pipe with a nonlinear viscoelastic material. Therefore, this is a very important step of the Finite Element Analysis. The selected element type must be compatible with layers and nonlinear properties.

For element type, Solid 186 has been used. for the viscoelastic material. This element type is selected due to its compatibility with nonlinear material and good mesh generation properties.

Preprocessor > Element Type > Add > Solid > 20node 186

### 3.3.2.6 Meshing:

It is the way of discretizing the area with the help of selected element type. The area after meshing has nodes and elements. Following are the steps involved in meshing:

Preprocessor > Mesh > Mesh Tools

Figure 3.8 shows how meshing of the pipe with the selected element “20 node 186” has been performed. Here, Quadrilateral shape and free meshing method have been used to generate uniform meshing through the cylinder volume. SOLID186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element supports plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials.

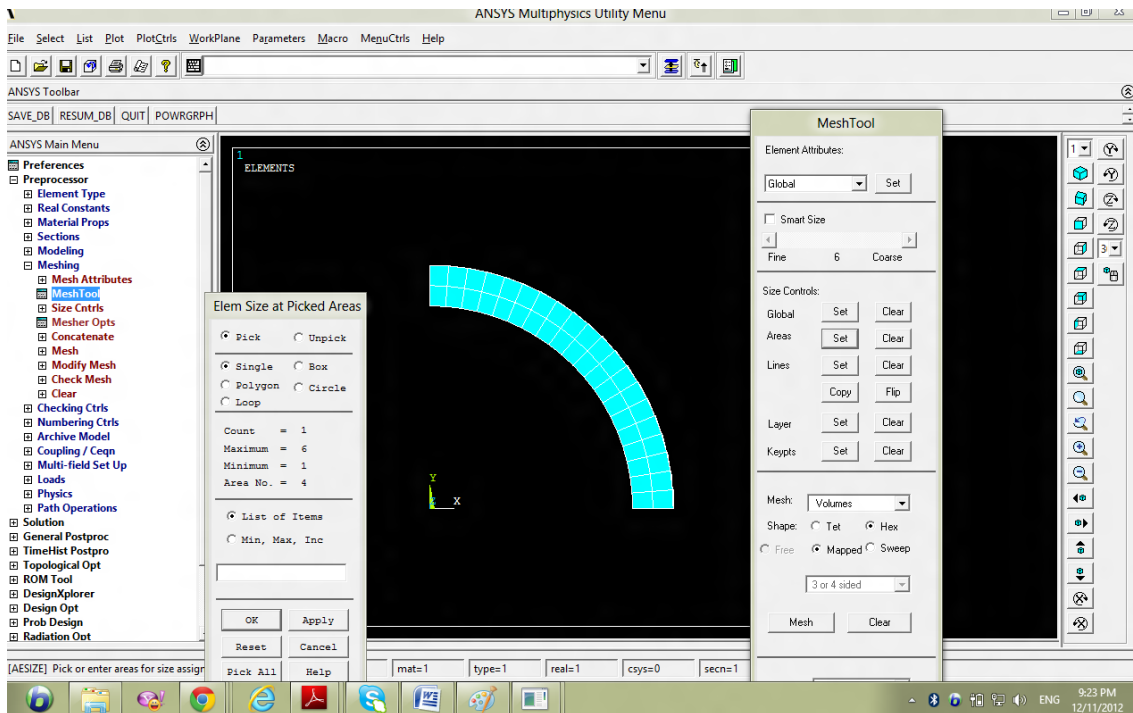


Figure 3.8: Meshing area using Meshing Tools

### 3.3.3 Solution

In this part boundary condition and solve condition are defined:

#### 3.3.3.1 Applying loads/ constraints:

Following are the steps to be followed to constrain pipe (partial cylinder).Hence, constraint a degrees of freedom equal to zero to its front and rear.Figure 3.9 & 3.10.

Preprocessor > Loads > Define Loads > Apply > Structural > Displacement > On Area

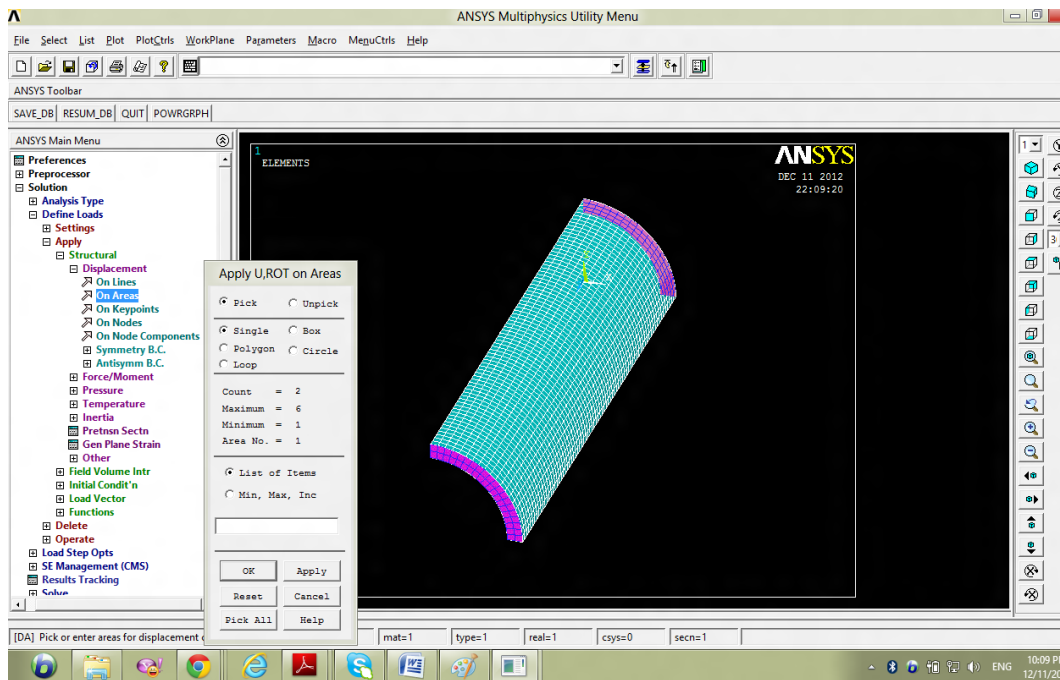


Figure 3.9: Defining boundary condition in ANSYS

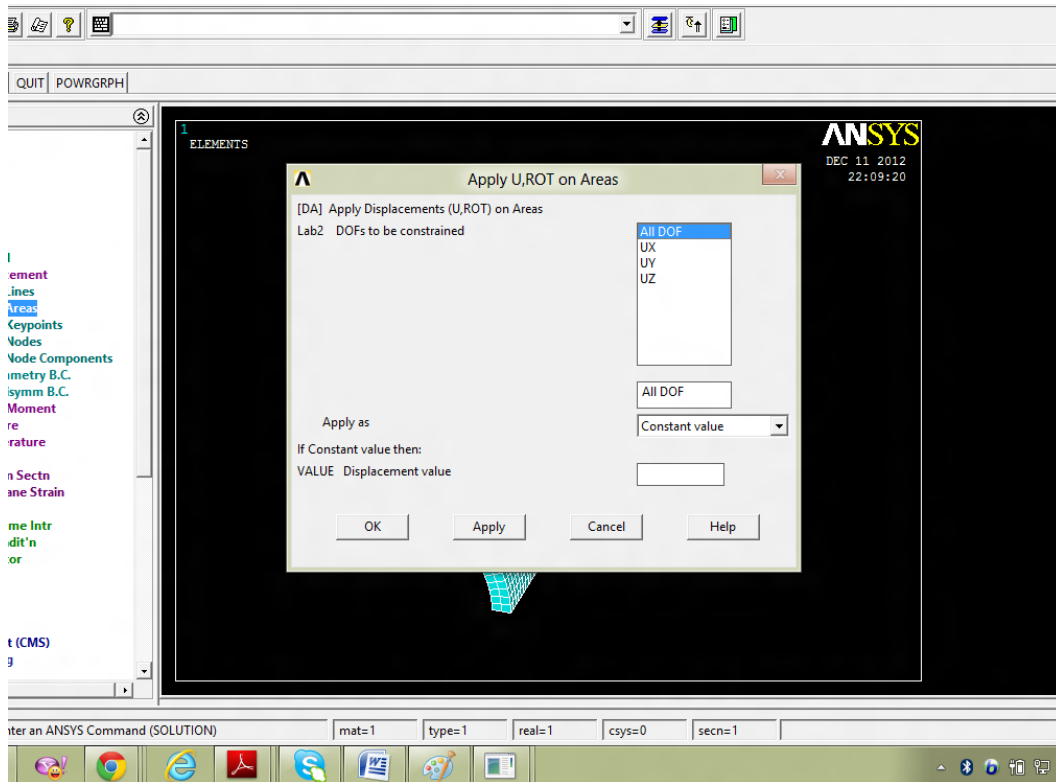


Figure 3.10: Defining boundary condition in ANSYS

This process are repeated for X and Y directions. To expose pipe by internal pressure which varied with time, tabular loads should be defined. There are essentially three steps in using tabular loads:

- Create a table parameter
- Fill in values of the table parameter
- Apply table parameter as a load to the model

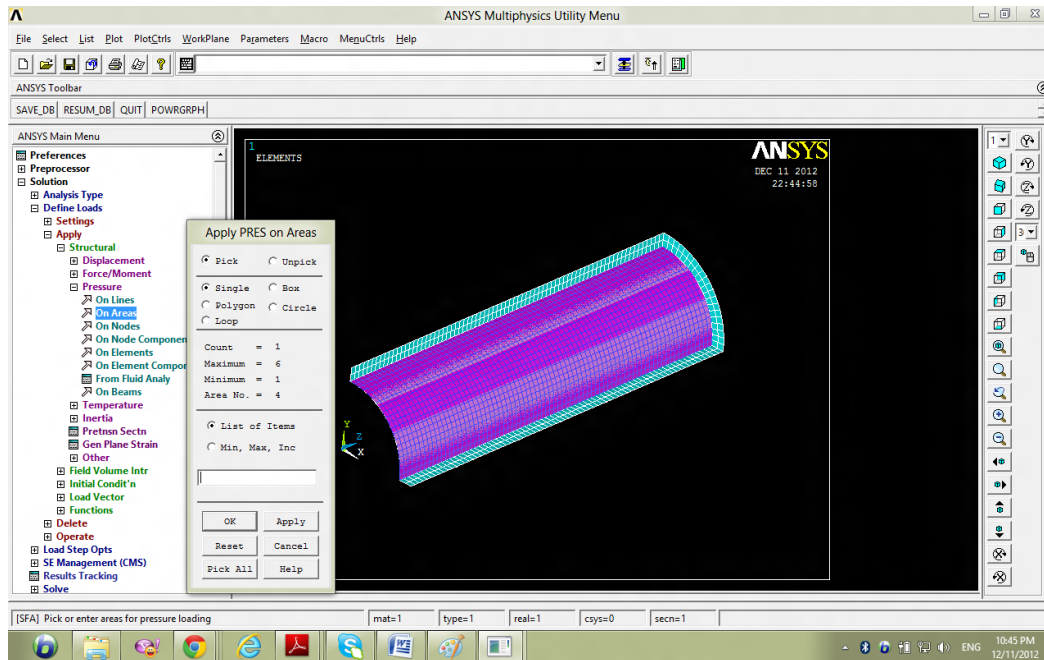


Figure3. 11: Defining the load in ANSYS

Utility Menu > Parameters > Array Parameters > Define/Edit ...> Click on [Add] Button

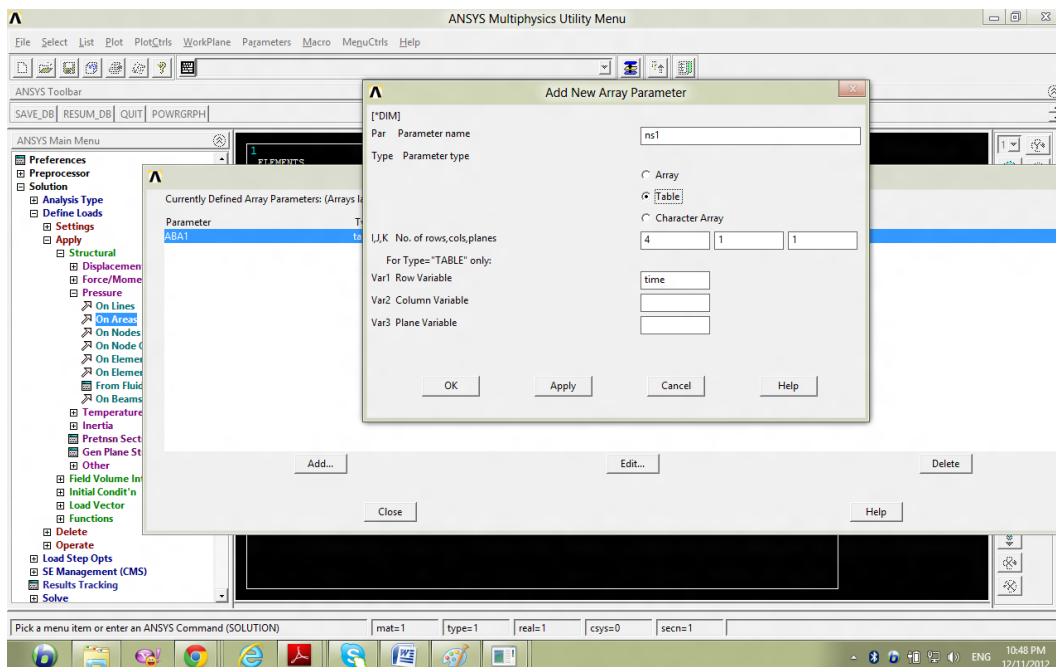


Figure 3.12: Defining the load in ANSYS

To confirm the applied loads by obtaining listings and displaying load symbols; then save the database. Figure 3.11 & 3.12

Utility Menu > List > Loads > DOF Constraints > on All Areas > Close

### **3.3.3.2 Solve**

To initiate the solution first need to turn non linear geometry situation on simply by;

NLGEOM,ON > Enter

And after that in Ansys main menu: Solution > Solve > Ok

### **3.3.4 General Postprocessing**

In this step we obtain the solution. Here, we can plot either the nodal solution or the element solution and “List results” and “Plot results” are the two important tools to display the results.

-ANSYS main Menu -General Postproc > Read Results > First Set > Plot Results > Deformed Shape > Ok



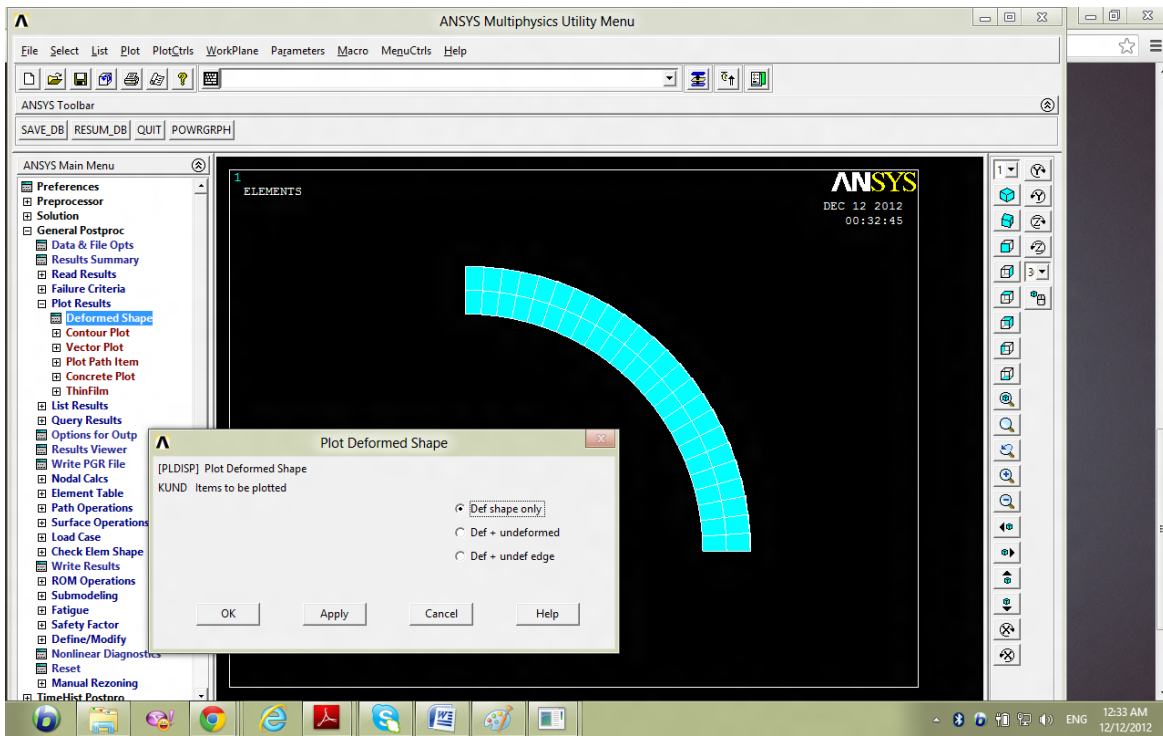


Figure 3.13: Plot deformed shape in ANSYS

-General Postproc >Plot Results > Nodal Solu > DOF solution > Displacement vector sum

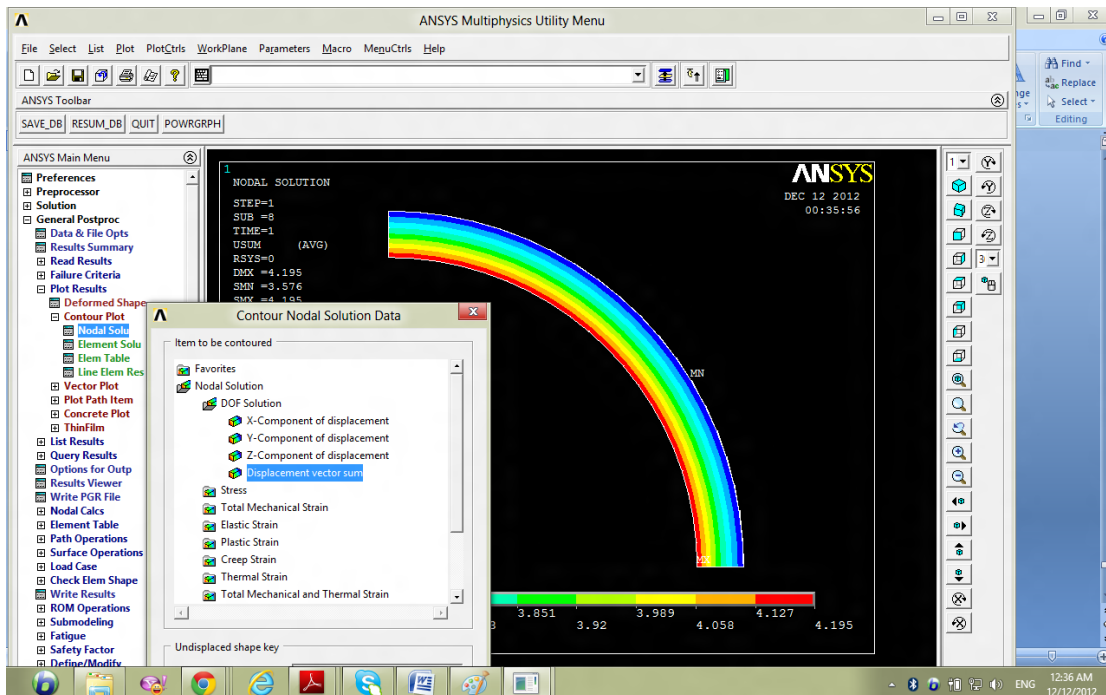


Figure 3.14: Plot results in ANSYS

-General Postproc >Plot Results > Nodal Solu > DOF solution > Elastic strain > 1st Principle elastic strain

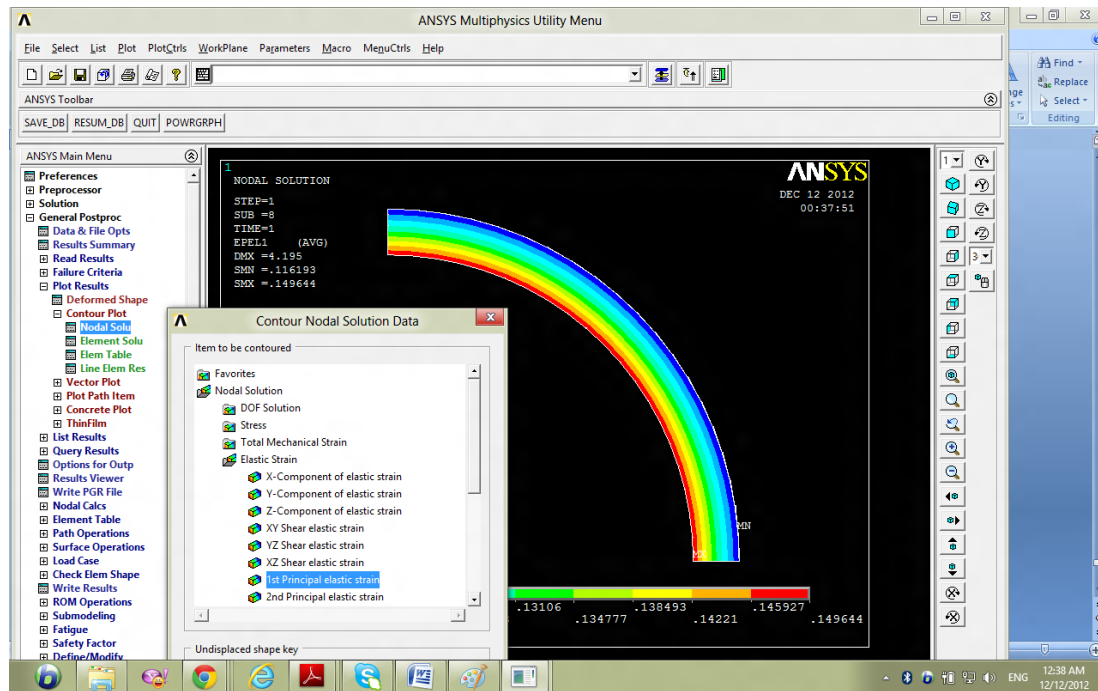


Figure 3.15: Plot results in ANSYS

## CHAPTER 4: RESULTS AND DISCUSSION

In previous chapter, NBR pipe with 3 different mentioned conditions.(Dry ,2 days and 5 days immersed in biodiesel ) simulated. Each condition has simulated in 5 different times.

In this chapter the results of simulation of non-swollen (Dry) condition for first chosen time step is showed in brief. In continue, the results of the all situations have demonstrated in figures 4.1 to 4.4. The numerical and some figured results attached in Appendix.

For first time step ( $t=0.3$ ), the results of stress and strain for different points of NBR pipe obtained by Ansys simulation are:

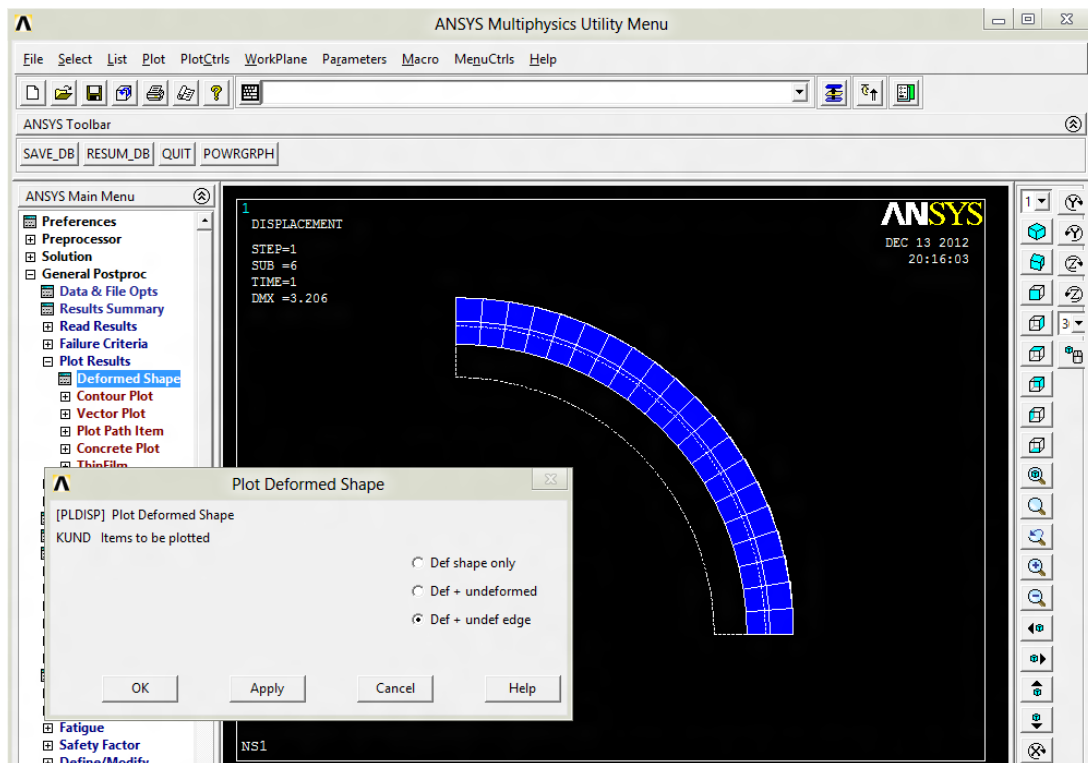
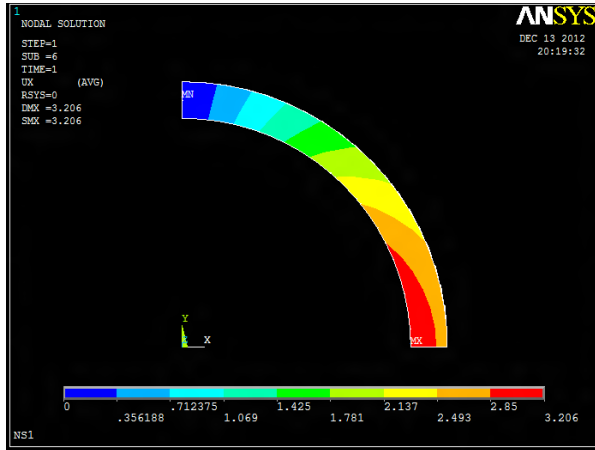


Figure 4.1: Deformed shape in result

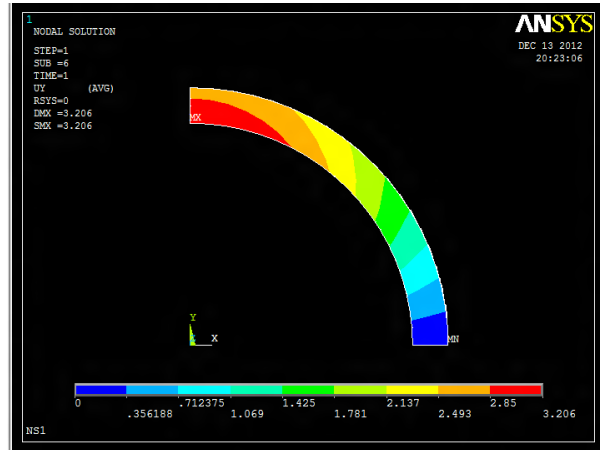
## 4.1 Displacement Analysis

Displacement Vector sum is the affection of a strains in 3 directions(X, Y and Z).As it seen in Figure 4.2, The highest displacement regard to initially position occur in inner surface of pipe.

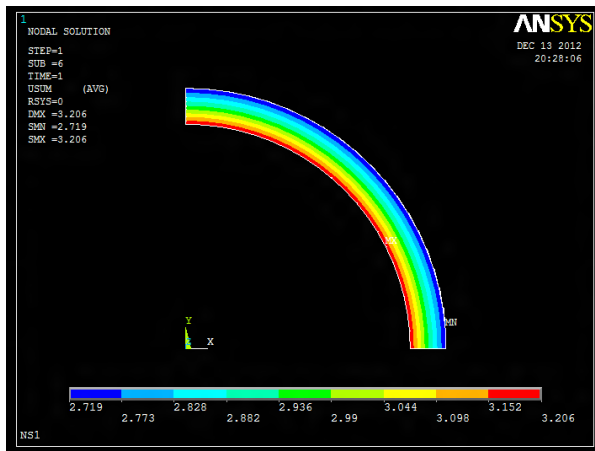
For DOF solution:



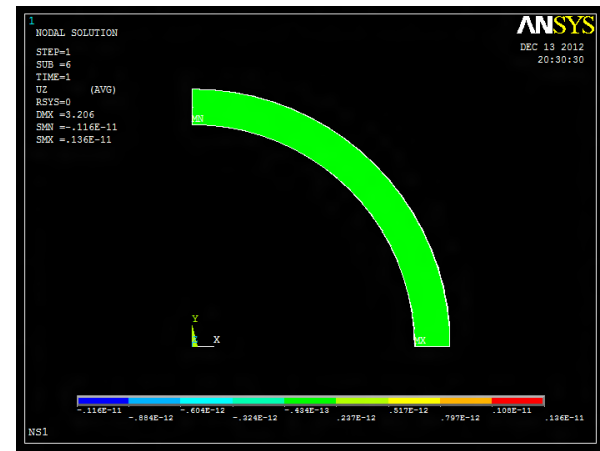
X-component of Displacement



Y-component of Displacement



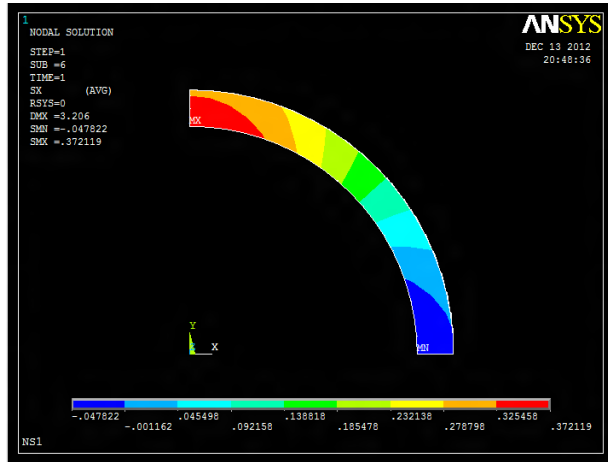
Displacement Vector Sum



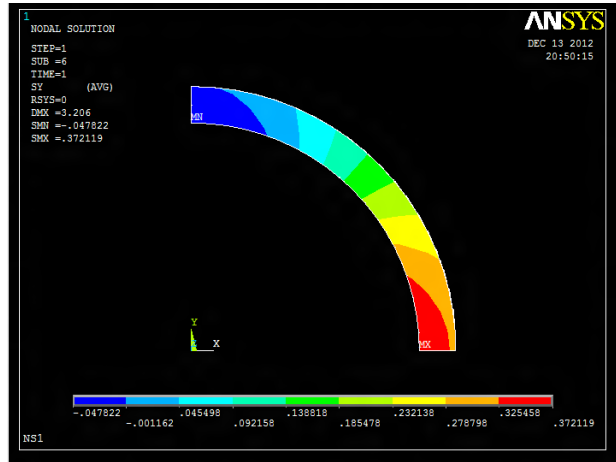
Z-component of Displacement

Figure 4.2: Displacements in different directions

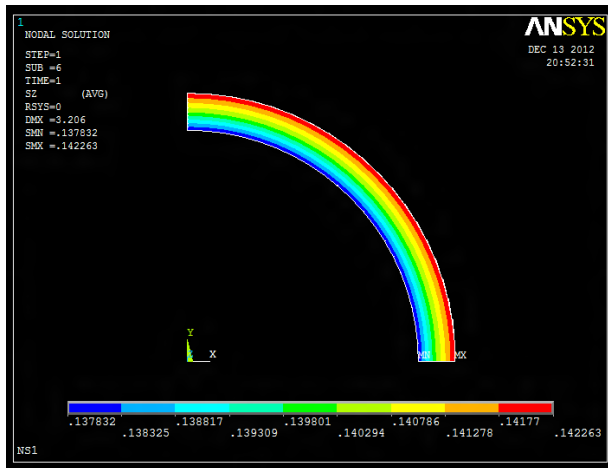
## 4.2 Stress Analysis



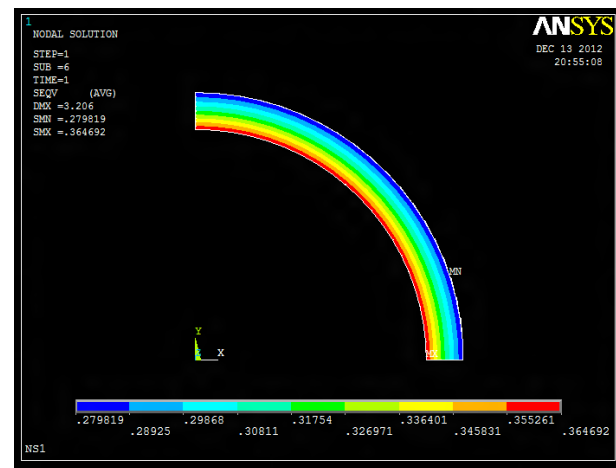
X-component of stress



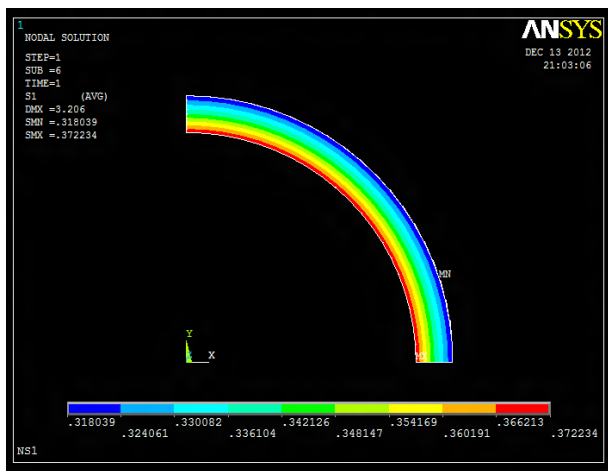
Y-component of stress



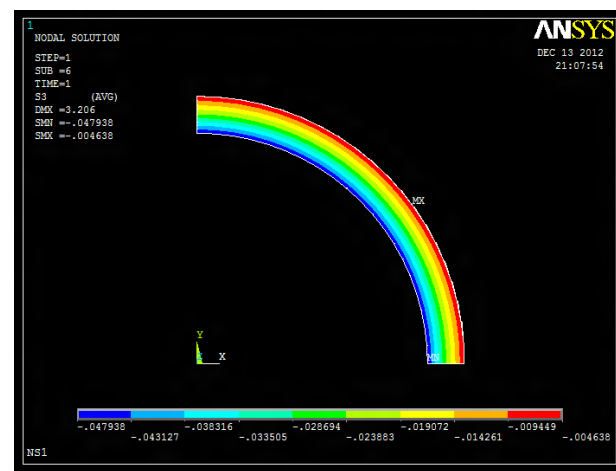
Z-component of stress



von Mises stress



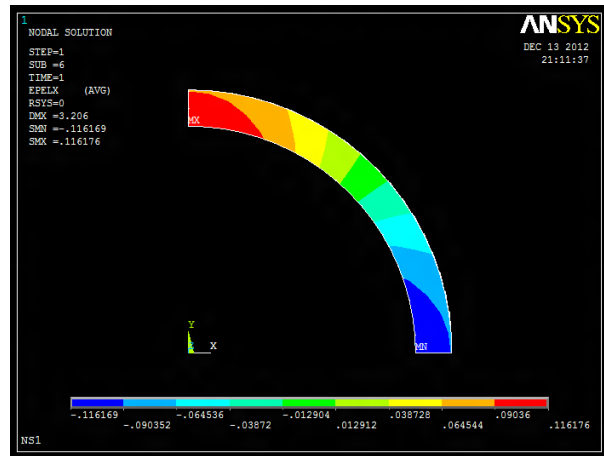
1<sup>st</sup> principal of stress



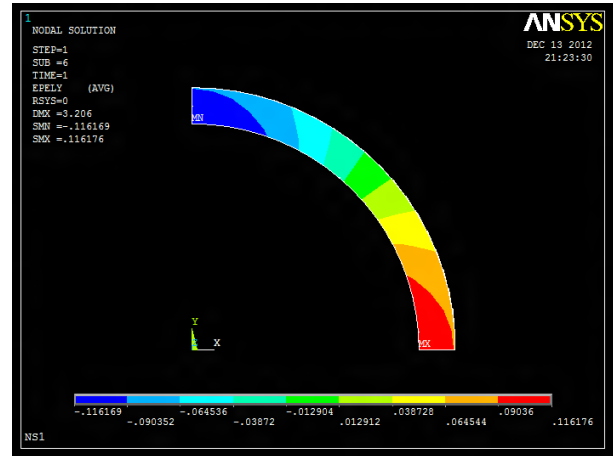
3<sup>rd</sup> principal of stress

Figure 4.3: Stress in different directions

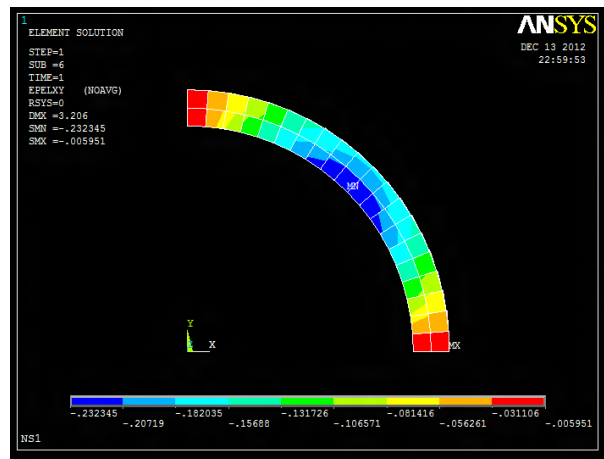
### 4.3 Strain Analysis :



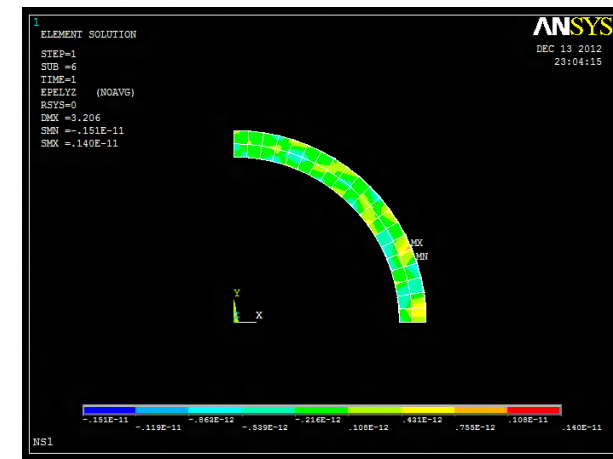
X-component of elastic strain



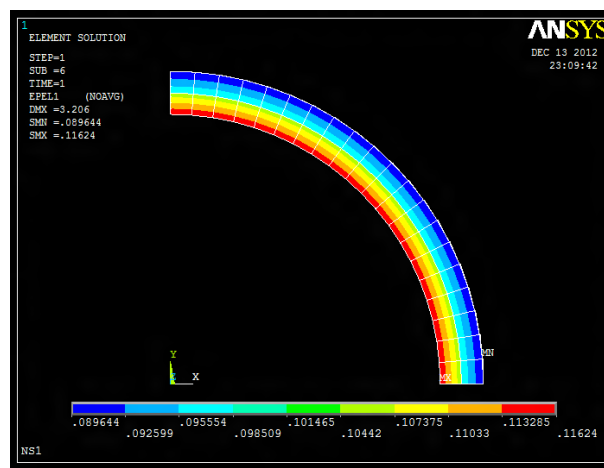
Y-component of elastic strain



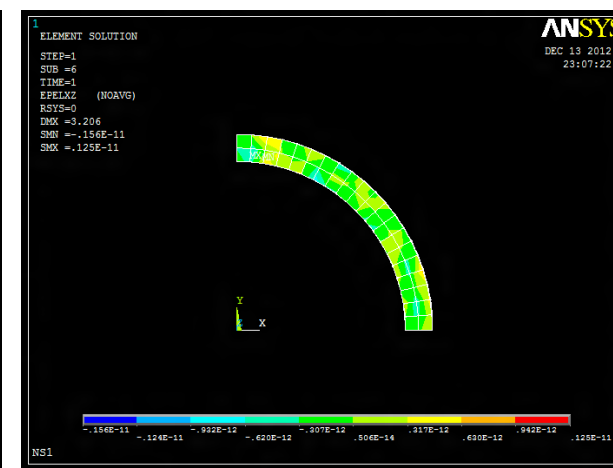
XY Shear elastic strain



YZ Shear elastic strain



1<sup>st</sup> principal of elastic strain

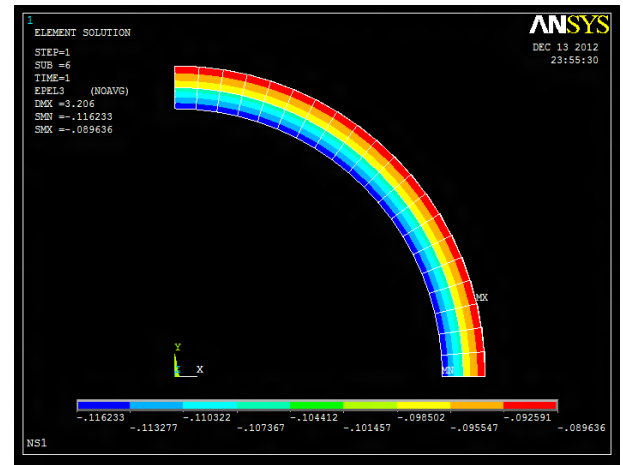


XZ Shear elastic strain

Figure 4.4: Elastic strain in different directions



2<sup>st</sup> principal of elastic strain



3<sup>st</sup> principal of elastic strain

Figure 4.5: Elastic strain in different directions

In this chapter mostly focused on analysis on Displacement vector sum and 1<sup>st</sup> principal elastic strain to find out different situation regard to time. The effect of the swelling on the time-dependent behavior of the elastomeric materials can be probed with various displacements illustrated in the inset of Figure 4.6

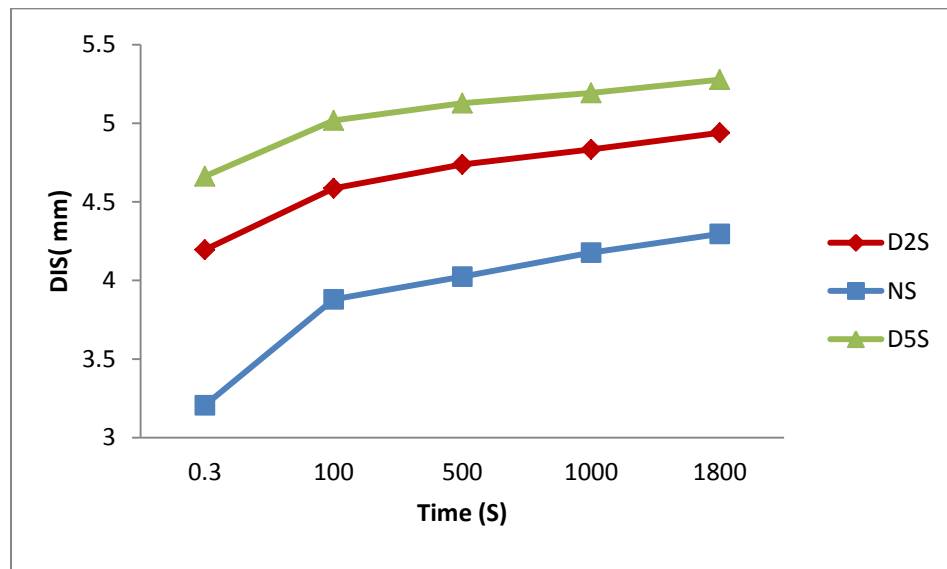


Figure 4.6: Displacement vector sum

Also if the 1<sup>st</sup> principal elastic strain results are showed in a Figure 4.7:

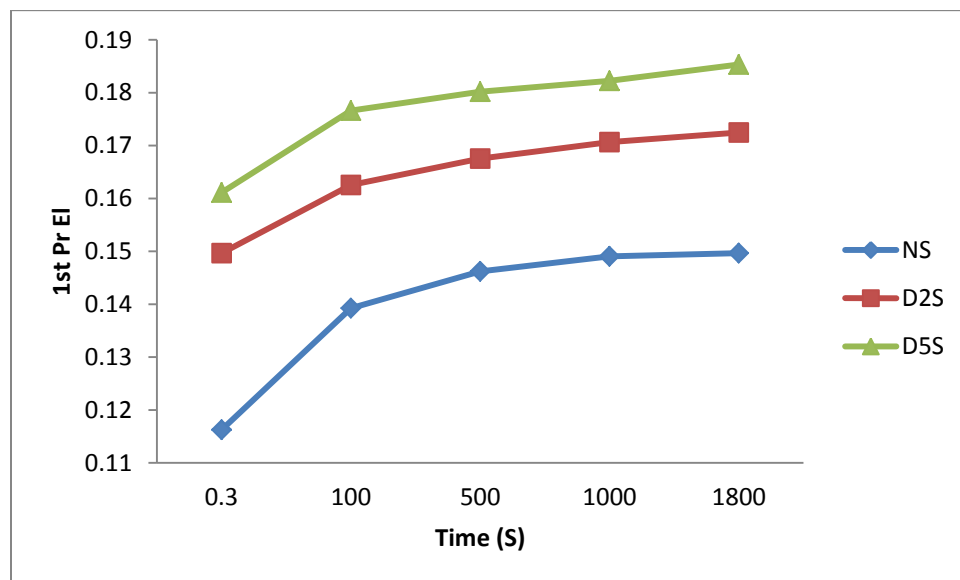


Figure 4.7: 1<sup>st</sup> principal elastic strain



To find the creep, each point should compare to other points. More investigation on creep is showed in figure 4.8 , where the normalized displacement vector sum an 1<sup>st</sup> principal elastic strain is plotted as a function of their changes for different time under internal pressure. These two figures the normalized displacement is defined by ratio between the value of displacement during the creep in specific time, Dis(s) and the beginning displacement achieved in t=0.3.

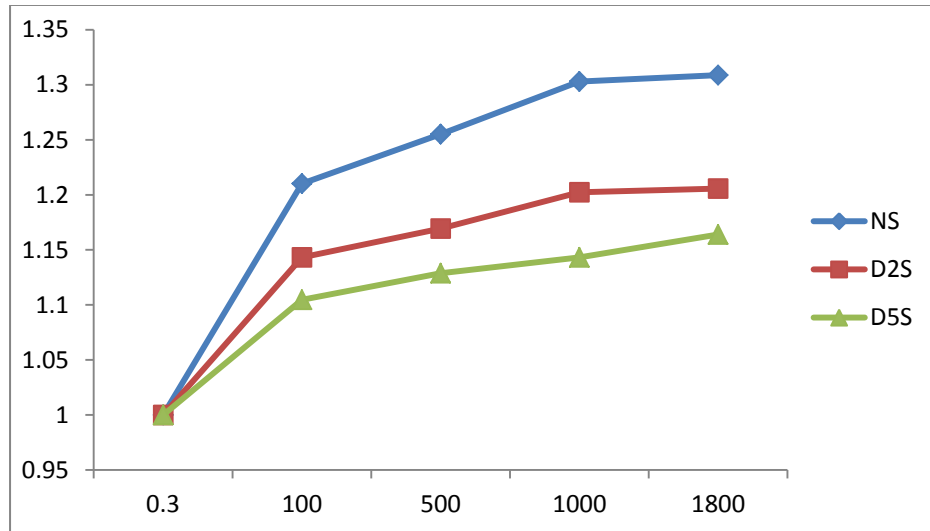


Figure 4.8: Normalized Displacement vector sum

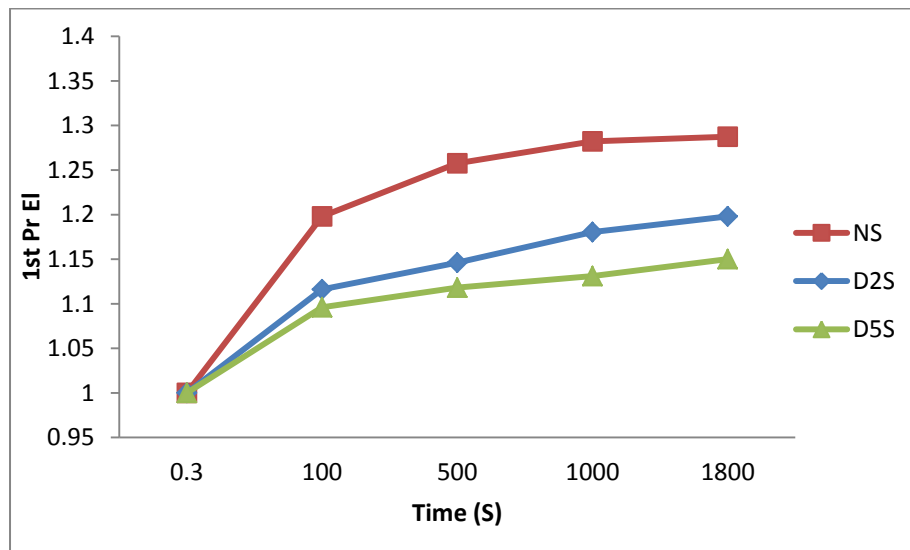


Figure 4.9: Normalized 1<sup>st</sup> principal elastic strain

According to Figures 4.8 & 4.9 which show same trends, it is obvious swollen NBR under loading becomes closer to the equilibrium state than dry (not swollen) NBR. Indeed, the creep is found to be smaller in the case of swollen rubber than the one in dry one. Also higher rate of swelling in NBR leads the viscous (time-dependent) contribution becomes smaller to the mechanical response.

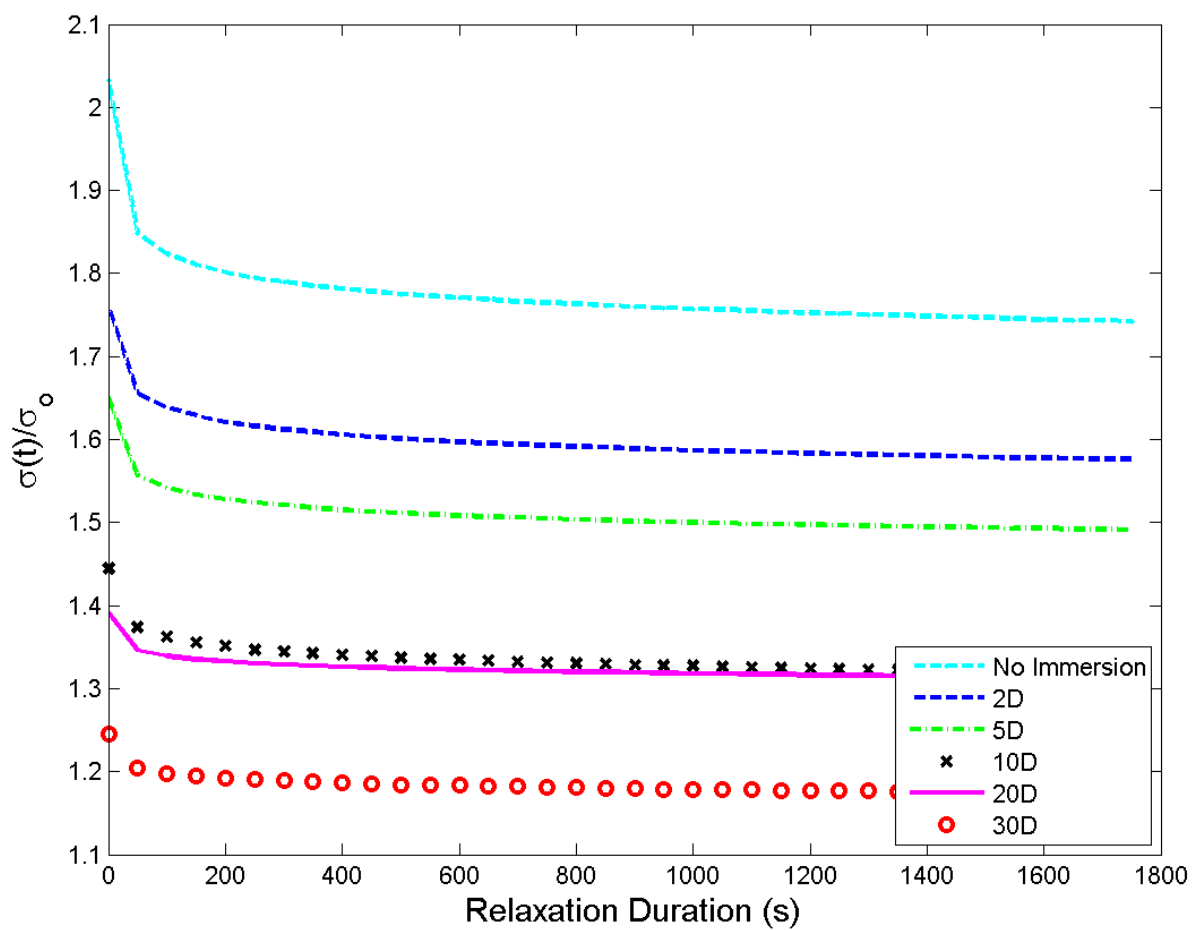
## ***CHAPTER5: CONCLUSION***

In this dissertation, the numerical simulation of creep in swollen elastomeric pipe was conducted. The simulation was performed using finite element code ANSYS. During simulation, it was assumed that both dry and swollen elastomers to be visco-hyperelastic, isotropic and incompressible. Two swelling levels were considered. The shear relaxation modulus need for the simulation was obtained from the experimental observation conducted by other student.

When a polymer is subjected to a constant load, it deforms quickly to a strain roughly predicted by its stress-strain modulus, and then continues to deform slowly with time-dependant response until rupture or yielding causes failure, sometimes described as the primary, secondary and tertiary phase of creep. It was found that the higher rate of swelling causes relatively higher deformation on short internal pressure durations. But in continue and longer internal pressure duration, higher rate of swelling causes less creep deformation .The diffusion of solvent into rubber appeared to reduce the strength of rubber. The long-term properties are more useful, the short-term stress-strain data is of little practical value in actual designing the part, since such data does not take into account the effect of long-term loading on Elastomers. However, the amount of the creep in swollen rubbers was found to be significantly lower than that in dry rubbers.

Finally, it is to note that these simulations focused only on the stress due to internal pressure in swollen pipe. Further investigations on the effect of other stress states on the swelling and the resulting creep are needed.

## APPENDIX



Shear moduli are collected from examples conducted by other students.(d=0.0001)

		mm	Time (s)	Displacement Vector Sum	1st Principal Elastic Strain	2nd Principal Elastic Strain	3th Principal Elastic Strain	Normalize displacement	Normalize 1st principal Elastic Strain
Not	NS1	0.9	0.3	3.206	0.11624	3.206	-0.116233	1	1
	NS2	0.7789	100	3.88	0.13926	3.88	-0.139252	1.2102	1.19804
	NS3	0.75	500	4.024	0.146191	4.024	-0.146183	1.2551	1.25766
Swollen	NS4	0.7389	1000	4.177	0.149045	4.177	-0.149036	1.3029	1.2822
	NS5	0.7366	1800	4.296	0.14965	4.296	-0.149642	1.3425	1.2874
Day 2	NS1	0.7368	0.3	4.195	0.149644	4.195	-0.149635	1	1
	NS2	0.6904	100	4.586	0.162539	4.586	-0.162531	1.0932	1.0862
	NS3	0.675	500	4.738	0.167527	4.738	-0.167518	1.1294	1.1195
Swollen	NS4	0.6669	1000	4.834	0.170648	4.834	-0.17064	1.1523	1.1521
	NS5	0.6622	1800	4.94	0.17246	4.94	-0.172452	1.1776	1.1776
Day5	NS1	0.6047	0.3	4.663	0.161132	4.663	-0.161124	1	1
	NS2	0.6504	100	5.018	0.176606	5.018	-0.176598	1.07613	1.096
	NS3	0.641	500	5.128	0.180176	5.128	-0.180167	1.0997	1.1182
Swollen	NS4	0.6357	1000	5.193	0.182255	5.193	-0.182246	1.1137	1.1311
	NS5	0.6281	1800	5.278	0.185322	5.278	-0.185314	1.1319	1.1501

*Results of NBR states(Dry , 2 days and 5 days swollen) in different times.*

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